Unit Rates

Common Core State Standards:
6.RP.2, 6.RP.3.b

Getting the Idea

A **rate** is a ratio that compares two quantities with different units of measure. Some examples of rates are shown below:

- Miles per gallon: 540 miles on 18 gallons of gas, $\frac{540 \text{ miles}}{18 \text{ gallons}}$
- Cost: \$3.60 for 4 pounds, or $\frac{\$3.60}{4 \text{ pounds}}$
- Pay rate: \$285 for 30 hours, or $\frac{\$285}{30 \text{ hours}}$

Rates are often given as a **unit rate**, which is a rate in which the second measure is 1 unit. Each of the rates listed above can be simplified as unit rates.

- Miles per gallon: $\frac{540 \text{ miles}}{18 \text{ gallons}} = \frac{30 \text{ miles}}{1 \text{ gallon}}$
- Cost: $\frac{\$3.60}{4 \text{ pounds}} = \frac{\$0.90}{1 \text{ pound}}$
- Pay rate: $\frac{\$285}{30 \text{ hours}} = \frac{\$9.50}{1 \text{ hour}}$

In general, for every ratio a:b, the corresponding unit rate is $\frac{a}{b}$, where $b \neq 0$.

For example, if there are 4 cups of cranberry juice to every 5 cups of orange juice in a punch recipe, the ratio of cranberry juice to orange juice is 4:5, or $\frac{4}{5}$. That means that there is $\frac{4}{5}$ cup of cranberry juice for every 1 cup of orange juice. You can see this mathematically by multiplying each quantity by 5:

$$\frac{\frac{4}{5}}{1} = \frac{\frac{4}{5} \times 5}{1 \times 5} = \frac{4}{5}$$

Example 1

A recipe for trail mix uses 5 ounces of mixed nuts, 6 ounces of dried fruit, and 4 ounces of granola. How many ounces of granola are there for every ounce of dried fruit?

Strategy **Write a ratio. Then find the unit rate.**

Step 1 Write the ratio of granola to dried fruit.

For every 4 ounces of granola, there are 6 ounces of dried fruit.

The ratio of granola to dried fruit is 4:6, or $\frac{4}{6}$.

In simplest form, $\frac{4}{6} = \frac{2}{3}$.

Step 2 Interpret the ratio as a unit rate.

The ratio 2:3 means that there is $\frac{2}{3}$ ounce of granola for every ounce of dried fruit.

Step 3 Check your work.

Multiply by 6.

$$\frac{\frac{2}{3}}{1} = \frac{\frac{2}{3} \times 6}{1 \times 6} = \frac{4}{6}$$

For every 4 ounces of granola, there are 6 ounces of dried fruit.

Solution **There is $\frac{2}{3}$ ounce of granola for each ounce of dried fruit.**

To find a unit price, identify the quantities you want to compare and write a rate. Then simplify the rate to find the unit price.

Example 2

Mr. Wilson spent $252 to stay 3 nights at Pavia Pavilions. At that rate, how much will he spend to stay 7 nights?

Strategy **Find the unit price. Then multiply by 7 nights.**

Step 1 Find the rate.

The rate is $252 for 3 nights, or $\frac{252}{3}$.

Step 2 Find the unit rate, or unit price.

Divide 252 by 3 to find the price for one night.

```
    84
 3)252
  -24
    12
   -12
     0
```

The unit price is $84 per night.

Step 3 Multiply the unit price by 7.

$7 \times 84 = 588$

Solution **Mr. Wilson will spend $588 to stay 7 nights at Pavia Pavilions.**

In Example 2, you could also have set up equivalent ratios to solve the problem. Let x represent the cost of staying 7 nights.

$$\frac{252}{3} = \frac{x}{7}$$

$3 \times x = 252 \times 7$ ← Cross multiply.

$3x = 1{,}764$ ← Divide both sides by 3.

$x = 588$

A common use of rate is the speed formula $r = \frac{d}{t}$, or rate $= \frac{\text{distance}}{\text{time}}$.

Example 3

A train is traveling at a constant speed of 45 miles per hour. How far will the train travel in 2.5 hours?

Strategy **Use the speed formula.**

Step 1 Substitute the known values in the speed formula.

$r = \frac{d}{t}$

$45 = \frac{d}{2.5}$ or $\frac{45}{1} = \frac{d}{2.5}$

Step 2 Find an equivalent fraction for $\frac{45}{1}$ with a denominator of 2.5.

$\frac{45}{1} = \frac{45 \times 2.5}{1 \times 2.5} = \frac{112.5}{2.5}$

$\frac{45 \text{ miles}}{1 \text{ hour}} = \frac{112.5 \text{ miles}}{2.5 \text{ hours}}$

Solution **The train will travel 112.5 miles in 2.5 hours.**

You can rewrite the speed formula $r = \frac{d}{t}$ to solve for either distance, d, or time, t.

If $r = \frac{d}{t}$, then $d = r \times t$.

If $r = \frac{d}{t}$, then $t = \frac{d}{r}$.

In Example 3, you could have used the formula $d = r \times t$ to solve the problem.

$d = r \times t$

$d = 45 \times 2.5$

$d = 112.5$

Coached Example

Tanya walked 15 laps on an indoor track in 30 minutes. What was Tanya's average speed in laps per minute?

The speed formula is $r =$ _____.

The distance is ______ laps.

The time is ______ minutes.

Substitute the known values into the speed formula.

$r =$ _____.

Simplify the fraction.

$r =$ _____

Tanya's average speed was ______ laps per minute.