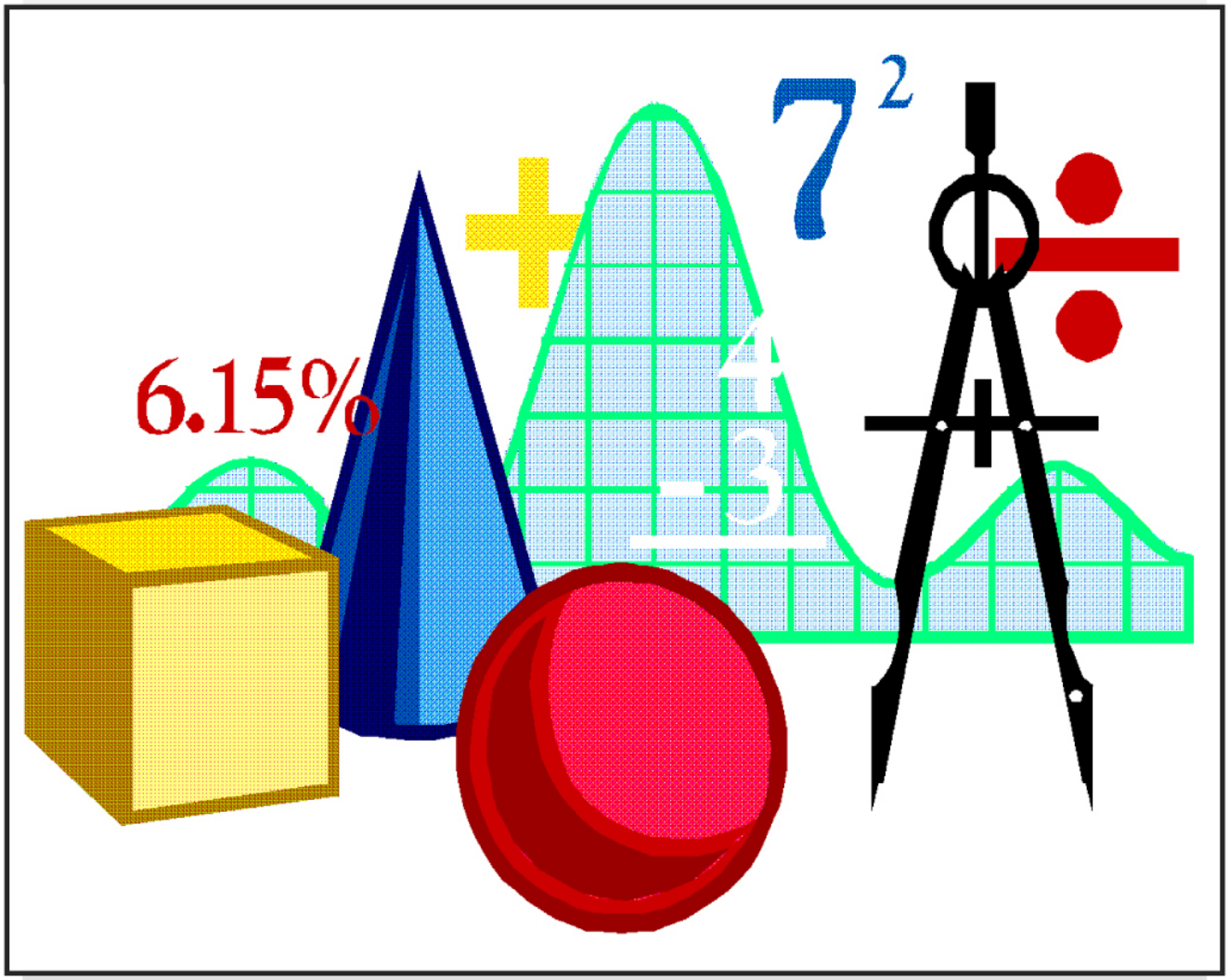
Curriculum Guide
to the Alabama Course of Study

Mathematics
Grades K-12



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CURRICLUM GUIDE TO THE ALABAMA COURSE OF STUDY: MATHEMATICS

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PREFACE

The *Curriculum Guide to the Alabama Course of Study: Mathematics* provides prerequisite and enabling skills that lead to learning grade-level academic standards. The curriculum guide can be used to assist students in learning content in smaller increments, catching up on content they may have missed in previous years, and/or reviewing content related to grade-level academic standards.

The 2011-2012 Curriculum Guide to the Alabama Course of Study Mathematics Task Force utilized the 2010 *Alabama Course of Study: Mathematics*. In addition, the Task Force reviewed past copies of the *Curriculum Guide to the Alabama Course of Study: Mathematics*. Members of the Task Force used their academic content knowledge and experiential knowledge related to students with and without disabilities to produce this resource for closing the gap between grade-level content and students' instructional levels.

ACKNOWLEDGMENTS

This document was developed by the 2011-2012 Curriculum Guide to the Alabama Course of Study Mathematics Task Force composed of elementary, middle school, high school, and college educators. The Task Force began the document during the summer of 2011.

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Introduction

The *Curriculum Guide to the Alabama Course of Study: Mathematics* is a companion document to the *2010 Alabama Course of Study: Mathematics*, for Grades K-12. Content standards contained within the course of study document may be accessed on the Alabama State Department of Education (ALSDE) Web site at www.alsde.edu. On the home page, click on Sections, scroll down to select Curriculum and Instruction. Click on Publications, scroll down to Courses of Study, and click on Mathematics.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local school systems may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

The *Curriculum Guide to the Alabama Course of Study: Mathematics* prepares students for study of the grade-level and course content standards through the teaching of prerequisite and enabling skills necessary for learning each content standard. This allows students to work toward grade-level and course content standards while working at individual ability levels. By identifying the prerequisites and enabling skills for each standard, teachers may plan instruction to address the achievement gap experienced by some students while still working with all students toward achievement of the same standards.

The *Curriculum Guide to the Alabama Course of Study: Mathematics* may be accessed at alex.state.al.us/specialed/curriculum.html.

Educators are encouraged to use the curriculum guide to:

- Develop lesson plans.
- Plan for Problem Solving Teams (PSTs) and Response to Intervention/Response to Instruction (RTI).
- Develop Individual Educational Programs (IEPs).
- Prepare for collaborative teaching.
- Design tutorials.
- Plan for instructional grouping.
- Plan for parent information and conferences.
- Develop curriculum-based assessments.
- Prepare for state assessments.

Organization of the Curriculum Guide

The organizational components of this guide include standards, instructional objectives, and examples.

Content standards are statements that define what all students should know and be able to do at the conclusion of a grade level or course. Content standards contain minimum required content and complete the phrase "*Students will*."

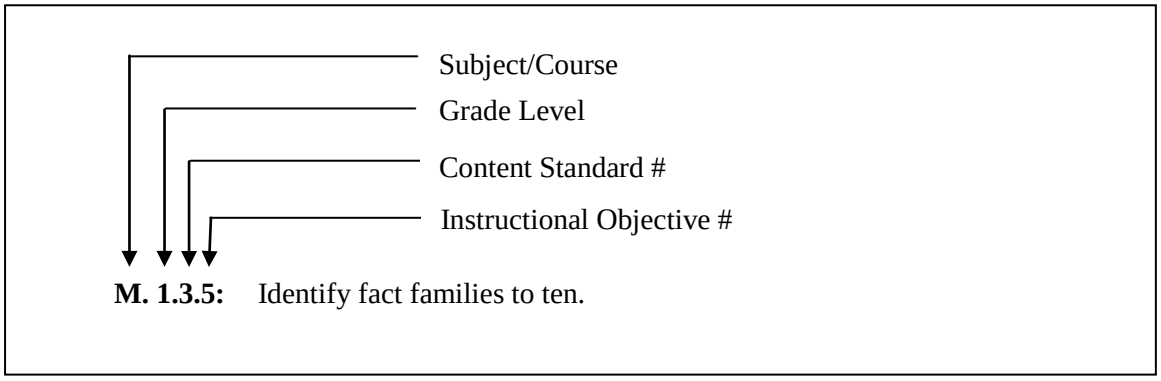
Content standards for a grade level or course are clearly written, reasonable, measurable, developmentally appropriate, and sufficiently rigorous to enable Alabama students to achieve at levels comparable to other students in the nation and the world. They also provide proportional emphasis to the essential knowledge, skills, and processes of a given grade level or course.

Instructional objectives divide the standards into smaller instructional units that serve as foundational skills for the standards. Instructional objectives are useful in lesson planning, classroom instruction, and Individualized Education Program (IEP) development. Utilization of instructional objectives facilitates having all students working toward grade-level standards while also working at individual ability levels.

Instructional objectives in Algebra I and Geometry preceded by a diamond shape (♦ or ◊) indicate content required for earning course credit for the *Alabama Occupational Diploma* (AOD) in Grades 9-12. Algebraic Concepts (Appendix B) is a stand-alone AOD course for students who need additional instruction prior to Algebraic Essentials A, the Alabama Occupational Diploma course that is aligned to Algebra I A.

Instructional objectives within this document are numbered according to grade level, content standard number, and the order in which the instructional objective is listed.

The system for numbering **Mathematics Objective 1. 3. 5.,** for example, is based upon the following:



Examples clarify certain content standards, instructional objectives, and/or their components. They are illustrative but not exhaustive.

Directions for interpreting the minimum required content are found in Appendix A, page 174.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices are based on important "processes and proficiencies" that have longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report, *Adding It Up*: *Helping Children Learn Mathematics*. These proficiencies include adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight Standards for Mathematical Practice are listed below along with a description of behaviors and performances of mathematically proficient students.

Mathematically proficient students:

1. **Make sense of problems and persevere in solving them.** These students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. These students consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to obtain the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solve complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships. One is the ability to *decontextualize*, to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. The second is the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** These students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. These students justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments

that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until the middle or upper grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** These students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, students might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, students might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas and can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use these to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** These students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Mathematically proficient students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students

can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. These students also can pause and reflect for an overview and shift perspective. They can observe the complexities of mathematics, such as some algebraic expressions as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.** They notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As students work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details and continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The eight Standards for Mathematical Practice described on the previous pages indicate ways in which developing student practitioners of the discipline of mathematics increasingly must engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. It is important that curriculum, assessment, and professional development designers be aware of the need to connect the mathematical practices to the mathematical content standards.

The *Common Core State Standards for Mathematics*, also referred to as the Standards for Mathematical Content, are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect mathematical practices to mathematical content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, pause for an overview, or deviate from a known procedure to find a shortcut. Thus, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Practice and the Standards for Mathematical Content. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the necessary time, resources, innovative energies, and focus to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

GRADE K

Students will:

Counting and Cardinality

Know number names and the count sequence.

1. Count to 100 by ones and by tens. [K-CC1]

Objectives:
M. K.1.1: Count to 50 by ones.
M. K.1.2: Count to 50 by tens.
M. K.1.3: Count to 20 by ones.
M. K.1.4: Count to 10 by ones.
M. K.1.5: Mimic counting by tens.
M. K.1.6: Mimic counting by ones.

2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). [K-CC2]

Objectives:
M. K.2.1: Count forward to 100 from a number between 2 and 50.
M. K.2.2: Count forward to 100 from a number over 50.
M. K.2.3: Count forward to 50 from a given number.
M. K.2.4: Count to 100 by ones.
M. K.2.5: Mimic counting to 100 by ones.

3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). [K-CC3]

Objectives:
M. K.3.1: Write numbers 0 to 10.
M. K.3.2: Match numerals to quantity 11 to 20.
M. K.3.3: Match numerals to quantity 0 to 10.
M. K.3.4: Recognize written numerals 0 to 20.
M. K.3.5: Demonstrate one to one correspondence for a group of objects 6 to 20.
M. K.3.6: Demonstrate one to one correspondence for a group of objects 0 to 5.
M. K.3.7: Trace numerals 0 to 20.
M. K.3.8: Make purposeful marks such as lines and circles.

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality. [K-CC4]

Objectives:
M. K.4.1: Define number and counting.
M. K.4.2: Identify correct number of objects for a given number up to 20.
M. K.4.3: Identify different size groups of objects up to 10.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. [K-CC4a]

Objectives:
M. K.4a.1: Count to 20 by ones.
M. K.4a.2: Mimic counting objects.

b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. [K-CC4b]

Objectives:
M. K.4b.1: Know that the last number tells how many when counting 0 to 5 objects.
M. K.4b.2: Mimic counting objects up to 20.
M. K.4b.3: Count to 20 by ones.
M. K.4b.4: Mimic counting to 20 by ones.

c. Understand that each successive number name refers to a quantity that is one larger. [K-CC4c]

Objectives:
M. K.4c.1: Define one larger/one more.
M. K.4c.2: Count objects in a group and identify total after adding one more.
M. K.4c.3: Count in sequential order.
M. K.4c.4: Mimic counting in sequential order.

5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. [K-CC5]

Objectives:
M. K.5.1: Define how many, all together, and in all.
M. K.5.2: Demonstrate one to one correspondence
Example: Point to only one object when counting, and stop counting when all objects have been touched.
M. K.5.3: Count to 20 by ones.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.) [K-CC6]

Objectives:
M. K.6.1: Define greater than, less than, and equal to.
M. K.6.2: Count to 20 by ones.
M. K.6.3: Count objects up to ten.

7. Compare two numbers between 1 and 10 presented as written numerals. [K-CC7]

Objectives:
M. K.7.1: Compare numbers 1 to 10 using objects.
M. K.7.2: Name numerals 1 to 10.
M. K.7.3: Identify numerals 1 to 10.
M. K.7.4: Count to 10 by ones.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

8. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problem. This applies wherever drawings are mentioned in the Standards.) [K-OA1]

Objectives:
M. K.8.1: Define addition as combining groups of objects.
M. K.8.2: Define subtraction as separating groups of objects.
M. K.8.3: Represent numbers with objects or drawings.
M. K.8.4: Separate sets with nine or fewer objects.
M. K.8.5: Combine objects to form sets up to nine.

9. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. [K-OA2]

Objectives:
M. K.9.1: Understand key words in addition and subtraction word problems.
Examples: all together, how many more, how many are left, in all
M. K.9.2: Represent numbers with objects or drawings.
M. K.9.3: Separate sets with nine or fewer objects.
M. K.9.4: Combine objects to form sets up to nine.

10. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). [K-OA3]

Objectives:
M. K.10.1: Identify plus, minus, and equal signs.
M. K.10.2: Match numerals to objects or drawings.
M. K.10.3: Identify numerals 1 to 10.
M. K.10.4: Count 0 to 10.

11. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. [K-OA4]

Objectives:
M. K.11.1: Write numerals from 0 to 10.
M. K.11.2: Represent a given numeral 1 to 10 with objects or drawings.
M. K.11.3: Count forward from a given number 1 to 10.
M. K.11.4: Model joining sets of objects to total 10.

12. Fluently add and subtract within 5. [K-OA5]

Objectives:
M. K.12.1: Decompose numbers up to 5 using objects or drawings.
M. K.12.2: Compose numbers up to 5 using objects or drawings.
M. K.12.3: Count backward from 5.
M. K.12.4: Count forward to 5.

Number and Operations in Base Ten

Work with numbers 11–19 to gain foundations for place value.

13. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. [K-NBT1]

Objectives:
M. K.13.1: Define ones and tens.
M. K.13.2: Match the number in the ones and tens position to a pictorial representation or manipulative of the value.
M. K.13.3: Add numbers 1-9 to ten to create teen numbers using manipulatives or place value blocks.
M. K.13.4: Count objects up to 10.

Measurement and Data

Describe and compare measurable attributes.

14. Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object. [K-MD1]

Objectives:
M. K.14.1: Define length and weight.
M. K.14.2: Explore objects in relationship to length and weight.

15. Directly compare two objects, with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference. [K-MD2]
    Example: Directly compare the heights of two children, and describe one child as taller or shorter.

Objectives:
M. K.15.1: Use vocabulary related to length and weight.
Examples: longer, shorter, heavier, lighter
M. K.15.2: Identify objects by length and weight.
Examples: shortest pencil, heaviest rock
M. K.15.3: Sort objects according to measurable attributes.

Classify objects and count the number of objects in each category.

16. Classify objects into given categories; count the number of objects in each category, and sort the categories by count. (Limit category counts to be less than or equal to 10.) [K-MD3]

Objectives:
M. K.16.1: Identify more and less when given two groups of objects.
M. K.16.2: Identify object attributes.
Examples: color, shape, size, texture, use
M. K.16.3: Count objects up to ten.
M. K.16.4: Count to 10 by ones.

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

17. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. [K-G1]

Objectives:
M. K.17.1: Recognize location and position.
Examples: above, below, beside, in front of, behind, next to
M. K.17.2: Identify cubes, cones, cylinders, and spheres.
M. K.17.4: Imitate actions to place items.
Examples: in, on, under
M. K.17.5: Match shapes.

18. Correctly name shapes regardless of their orientations or overall size. [K-G2]

Objectives:
M. K.18.1: Recognize shapes.
M. K.18.2: Sort shapes with different attributes.
Example: sort different size or color squares, circles, triangles, rectangles or hexagons

19. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). [K-G3]

Objectives:
M. K.19.1: Define two-dimensional and three-dimensional.
Example: two-dimensional shapes are flat, three-dimensional figures are solid
M. K.19.2: Sort flat and solid objects.
M. K.19.3: Explore two-dimensional shapes and three-dimensional figures.

Analyze, compare, create, and compose shapes.

20. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices or "corners"), and other attributes (e.g., having sides of equal length). [K-G4]

Objectives:
M. K.20.1: Define similar and different.
M. K.20.2: Use vocabulary related to two-dimensional shapes and three-dimensional figures.
Examples: vertices (corners), faces (flat surfaces), edges, sides, angles
M. K.20.3: Recognize vocabulary related to two-dimensional shapes and three-dimensional figures.

M. K.20.4: Identify two-dimensional shapes and three-dimensional figures.
M. K.20.5: Identify shapes.

21. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. [K-G5]

Objectives:
M. K.21.1: Recognize attributes of shapes.
M. K.21.2: Identify cubes, cones, cylinders, and spheres.
M. K.21.3: Identify squares, circles, triangles, rectangles, and hexagons.
M. K.21.4: Identify shapes in the environment.
M. K.21.5: Trace shapes.
M. K.21.6: Make purposeful marks such as lines and circles.

22. Compose simple shapes to form larger shapes. [K-G6]
Example: *"Can you join these two triangles with full sides touching to make a rectangle?"*

M. K.22.1: Combine shapes to fill the area of a given shape.
M. K.22.2: Decompose pictures made of simple shapes.
M. K.22.3: Recognize shapes.
M. K.22.4: Match pieces by color, image, or shape to complete a puzzle.

GRADE 1

Students will:

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Appendix A, Table 1.) [1-OA1]

Objectives:

M. 1.1.1: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
M. 1.1.2: Understand key words in addition and subtraction word problems.
Examples: sum, difference, all together, how many more, how many are left, in all
M. 1.1.3: Define subtraction as separating groups of objects, taking from, or taking apart.
M. 1.1.4: Define addition as combining groups of objects, adding to, or putting together.
M. 1.1.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.
M. 1.1.6: Represent numbers with objects or drawings.
M. 1.1.7: Use objects to combine and separate groups.

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. [1-OA2]

Objectives:

M. 1.2.1: Solve addition word problems with sums less than or equal to 10, e.g., by using objects or drawings to represent the problem.
M. 1.2.2: Understand key words in addition word problems.
Examples: sum, all together, how many more, in all
M. 1.2.3: Define addition as combining groups of objects, adding to, or putting together.
M. 1.2.4: Represent addition with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.
M. 1.2.5: Represent numbers with objects or drawings.
M. 1.2.6: Use objects to combine groups.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) [1-OA3]
   Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition)

Objectives:
M. 1.3.1: Define addition and subtraction.
M. 1.3.2: Recognize properties of operations.
M. 1.3.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M. 1.3.4: Apply signs +, -, = to actions of joining and separating sets.
M. 1.3.5: Identify fact families to ten.
M. 1.3.6: Recognize the value of zero.

4. Understand subtraction as an unknown-addend problem. [1-OA4]
   Example: Subtract 10 – 8 by finding the number that makes 10 when added to 8.

Objectives:
M. 1.4.1: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M. 1.4.2: Identify fact families to ten.
M. 1.4.3: Recall basic addition facts to ten.
M. 1.4.4: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). [1-OA5]

Objectives:
M. 1.5.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.
M. 1.5.2: Count forward and backward from a given number.
M. 1.5.3: Count to 20 by ones.

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). [1-OA6]

Objectives:

M. 1.6.1: Decompose numbers less than or equal to 10.
M. 1.6.2: Add and subtract within 5.
M. 1.6.3: Count forward and backward from a given number.
M. 1.6.4: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. [1-OA7]
   Example: Which of the following equations are true and which are false:
   6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2?

Objectives:

M. 1.7.1: Define true, false, and equal.
M. 1.7.2: Demonstrate equal using manipulatives or object drawings.
M. 1.7.3: Recall basic addition facts to ten.
M. 1.7.4: Recognize equation symbols in vertical and horizontal addition and subtraction problems.
M. 1.7.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. [1-OA8]
   Example: Determine the unknown number that makes the equation true in each of the equations, 8 + ? = 11, 5 = □ – 3, and 6 + 6 = □.

Objectives:

M. 1.8.1: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M. 1.8.2: Identify fact families as a relationship between addition and subtraction.
M. 1.8.3: Recall basic addition and subtraction facts to ten.
M. 1.8.4: Identify plus, minus, and equal signs.
M. 1.8.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

Number and Operations in Base Ten

Extend the counting sequence.

9. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. [1-NBT1]

Objectives:
M. 1.9.1: Write numerals from 0 to 20.
M. 1.9.2: Recognize numerals to 100.
M. 1.9.3: Match the numeral to the number objects or picture of objects.
M. 1.9.4: Count to 100 by ones.
M. 1.9.5: Count to 20 by ones.
M. 1.9.6: Identify and name numerals 1-9.
M. 1.9.7: Trace numerals 1-9.

Understand place value.

10. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: [1-NBT2]

    a. 10 can be thought of as a bundle of ten ones, called a "ten." [1-NBT2a]
    b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. [1-NBT2b]
    c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). [1-NBT2c]

Objectives:
M. 1.10.1: Match the number in the ones and tens position to a pictorial representation or manipulative of the value.
M. 1.10.2: Represent numbers with multiple models.
Examples: models – base ten blocks, number lines, linking cubes, straw bundles
M. 1.10.3: Count to 100 by tens.
M. 1.10.4: Count 10 objects.
M. 1.10.5: Count to 10 by ones.
M. 1.10.6: Name numerals 0 to 19.

11. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. [1-NBT3]

Objectives:
M. 1.11.1: Define greater than, less than and equal to.
M. 1.11.2: Demonstrate greater than, less than, and equal to using manipulatives, object drawings or numbers 0 to 10.
M. 1.11.2: Use comparison vocabulary.
Examples: greater than, equal to, and less than.
M. 1.11.3: Recognize symbols for greater than, less than and equal to.
M. 1.11.4: Determine the value of the digits in the ones and tens place.
M. 1.11.5: Identify sets with more, less or equal objects.
M. 1.11.6: Imitate creating sets of a given size.

Use place value understanding and properties of operations to add and subtract.

12. Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. [1-NBT4]

Objectives:
M. 1.12.1: Demonstrate regrouping, total sum, and solve using drawings and concrete models.
M. 1.12.2: Model written method for recording horizontal addition problems.
M. 1.12.3: Determine the value of the number in the ones and tens place.
M. 1.12.4: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.
M. 1.12.5: Represent numbers with multiple models.
Examples: models – base ten blocks, number lines, linking cubes, straw bundles
M. 1.12.6: Recall single-digit addition facts.

13. Given a two-digit number, mentally find 10 more or 10 less than the number without having to count; explain the reasoning used. [1-NBT5]

Objectives:
M. 1.13.1: Define more and less.
M. 1.13.2: Demonstrate conceptual understanding of adding or subtracting 10 using concrete models.
M. 1.13.3: Count backward from 100 by tens.
M. 1.13.4: Count forward to 100 by tens.
M. 1.13.4: Count to 100 by ones.

14. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used. [1-NBT6]

Objectives:
M. 1.14.1: Demonstrate conceptual understanding of subtraction using concrete models.
M. 1.14.2: Model written method for recording problems involving subtraction of 10 from multiples of 10.
M. 1.14.3: Count backward from 100 by tens.
M. 1.14.4: Count forward to 100 by tens.
M. 1.14.5: Mimic counting to 100 by tens.

Measurement and Data

Measure lengths indirectly and by iterating length units.

15. Order three objects by length; compare the lengths of two objects indirectly by using a third object. [1-MD1]

Objectives:
M. 1.15.1: Define length.
M. 1.15.2: Use vocabulary related to length.
Examples: longer, shorter, longest, shortest, taller
M. 1.15.3: Identify objects by length.
Examples: shortest pencil, tallest boy
M. 1.15.4: Sort objects according to length.
Example: sort short pencils from long pencils
M. 1.15.5: Explore objects in relationship to length.

16. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* [1-MD2]

Objectives:
M. 1.16.1: Describe gap and overlap.
M. 1.16.2: Describe what it means to measure using non-standard units.
M. 1.16.3: Model measuring using non-standard units.

Tell and write time.

17. Tell and write time in hours and half-hours using analog and digital clocks. [1-MD3]

Objectives:
M. 1.17.1: Describe the short hand as the hour hand and the long hand as the minute hand on an analog clock.
M. 1.17.2: Describe the first number as the hour, and the numbers after the colon as the minutes on a digital clock.
M. 1.17.3: Count to 30 by fives.
M. 1.17.4: Recognize numbers 1 to 12, and 30.
M. 1.17.5: Trace numerals 1 to 12, and 30.
M. 1.17.6: Associate digital and analog clocks with the measurement of time.

Represent and interpret data.

18. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. [1-MD4]

Objectives:
M. 1.18.1: Define more and less.
M. 1.18.2: Describe methods for representing data.
Examples: pictographs, tally charts, bar graphs, and Venn Diagrams
M. 1.18.3: Locate information on data displays.
M. 1.18.4: Classify objects into given categories; count the number of objects in each category, and sort the categories by count.
M. 1.18.5: Recognize different types of data displays.

Geometry

Reason with shapes and their attributes.

19. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. [1-G1]

Objectives:
M. 1.19.1: Define side, angle, closed and open.
M. 1.19.2: Describe attributes of shapes.
Examples: number of sides, number of angles
M. 1.19.3: Identify two-dimensional shapes.
M. 1.19.4: Sort two-dimensional shapes.
M. 1.19.5: Identify basic attributes.
Examples: color, shape, size

20. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as “right rectangular prism.”) [1-G2]

Objectives:
M. 1.20.1: Combine shapes to fill in the area of a given shape.
M. 1.20.2: Replicate composite shapes.
M. 1.20.3: Decompose pictures made of simple shapes.
M. 1.20.4: Name shapes.
Examples: square, circle, triangle, rectangle, and hexagon.
M. 1.20.5: Recognize shapes.

21. Partition circles and rectangles into two and four equal shares; describe the shares using the words *halves*, *fourths*, and *quarters*; and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. [1-G3]

Objectives:

M. 1.21.1: Define halves, fourths, quarters, whole, parts (shares) and equal.
M. 1.21.2: Demonstrate sharing situations to show equal smaller shares.
M. 1.21.3: Distinguish between equal and non-equal parts.
M. 1.21.4: Decompose pictures made of simple shapes.
M. 1.21.5: Identify squares, circles, triangles and rectangles.

GRADE 2

Students will:

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Appendix A, Table 1.) [2-OA1]

Objectives:

M. 2.1.1: Solve one-step addition and subtraction word problems with an unknown by using drawings and equations with a symbol for the unknown number to represent the problem.

M. 2.1.2: Understand key words in addition and subtraction word problems.
Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how many more, how many are left, in all

M. 2.1.3: Locate the unknown regardless of position.
Examples: start unknown, change unknown, and result unknown

M. 2.1.4: Apply signs +, -, = to actions of joining and separating sets.

M. 2.1.5: Add and subtract within 50, e.g., by using objects or drawings to represent the problem.

M. 2.1.6: Solve addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects or drawings to represent the problem.

M. 2.1.7: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

M. 2.1.8: Represent numbers with objects or drawings.

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies. (See standard 6, Grade 1, for a list of mental strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers. [2-OA2]

Objectives:

M. 2.2.1: Recall single – digit subtraction facts with minuends of 10 or less.

M. 2.2.2: Recall single – digit addition facts with sums up to 10.

M. 2.2.3: Apply addition and subtraction strategies.
Examples: doubles, doubles plus one, doubles minus one

M. 2.2.4: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. [2-OA3]

Objectives:
M. 2.3.1: Define pair, odd and even.
M. 2.3.2: Recall doubles addition facts with sums to 20.
M. 2.3.3: Apply signs + and = to actions of joining sets.
M. 2.3.4: Model written method for composing equations.
M. 2.3.5: Skip count by 2s.

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. [2-OA4]

Objectives:
M. 2.4.1: Distinguish between rows and columns.
M. 2.4.2: Use repeated addition to solve problems with multiple addends.
M. 2.4.3: Count forward in multiples from a given number.
Examples: 3, 6, 9, 12; 4, 8, 12, 16
M. 2.4.4: Recall doubles addition facts.
M. 2.4.5: Model written method for composing equations.

Number and Operations in Base Ten

Understand place value.

5. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: [2-NBT1]

   a. 100 can be thought of as a bundle of ten tens, called a "hundred." [2-NBT1a]
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). [2-NBT1b]

Objectives:
M. 2.5.1: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.
M. 2.5.2: Represent numbers with multiple concrete models.
Examples: concrete models – base ten blocks, number lines, linking cubes, straw bundles
M. 2.5.3: Count to 1000 by hundreds.
M. 2.5.4: Count to 100 by tens.
M. 2.5.5: Create groups of 10.
M. 2.5.6: Match the numeral in the ones and tens position to a pictorial representation or manipulative of the value.
M. 2.5.7: Match the numeral to the number of objects or picture of objects.

6. Count within 1000; skip-count by 5s, 10s, and 100s. [2-NBT2]

Objectives:
M. 2.6.1: Create a number pattern.
M. 2.6.2: Count backward from 100 by fives and tens.
M. 2.6.3: Count forward to 100 by fives and tens.
M. 2.6.4: Count to 100 by ones.

7. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. [2-NBT3]

Objectives:
M. 2.7.1: Identify zero as a place holder in two–digit and three–digit numbers.
M. 2.7.2: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.
M. 2.7.3: Identify the value of number in the ones, tens and hundreds place.
M. 2.7.4: Identify place value for ones, tens and hundreds.
M. 2.7.5: Read number names one through one hundred.
M. 2.7.6: Write numerals 1 to 100.
M. 2.7.7: Recognize number names one through twenty.
M. 2.7.8: Trace numerals 0 to 100.

8. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons. [2-NBT4]

Objectives:
M. 2.8.1: Define greater than, less than and equal to.
M. 2.8.2: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
M. 2.8.3: Arrange two – digit numbers in order from greatest to least or least to greatest.
M. 2.8.4: Identify zero as a place holder in two – digit and three – digit numbers.
M. 2.8.5: Model using >, =, and < symbols to record the results of comparisons of two two-digit numbers.
M. 2.8.6: Select numbers on a number line that are more than, less than or equal to a specified number.
M. 2.8.7: Match the words greater than, equal to and less than to the symbols >, =, and <.
M. 2.8.8: Determine the value of the digits in the ones and tens place.
M. 2.8.9: Identify sets with more, less or equal objects.

Use place value understanding and properties of operations to add and subtract.

9. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. [2-NBT5]

Objectives:

M. 2.9.1: Define regrouping, total, sum, difference and solve.
M. 2.9.2: Represent numbers with multiple models.
Examples: models – base ten blocks, number lines, linking cubes, straw bundles
M. 2.9.3: Recall single-digit addition and subtraction facts.
M. 2.9.4: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.
M. 2.9.5: Add and subtract within 5.
M. 2.9.6: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.

10. Add up to four two-digit numbers using strategies based on place value and properties of operations. [2-NBT6]

Objectives:

M. 2.10.1: Add within 100, including adding a two-digit number and a one-digit number and adding two two-digit numbers, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used.
M. 2.10.2: Add within 20, demonstrating fluency for addition within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; and creating equivalent but easier or known sums.
M. 2.10.3: Determine the value of the number in the ones, tens and hundreds place.
M. 2.10.4: Model written method for recording horizontal and vertical addition problems.
M. 2.10.5: Understand that the two digits of a two-digit number represent amounts of tens and ones.
M. 2.10.6: Match the number in the ones and tens position to a pictorial representation or manipulative of the value.

11. Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. [2-NBT7]

Objectives:

M. 2.11.1: Define regrouping, total, sum, difference and solve.
M. 2.11.2: Add and subtract two two-digit numbers with and without regrouping.
M. 2.11.3: Determine the value of the number in the ones, tens, hundreds and thousands place using concrete models or drawings and strategies based on place value.
M. 2.11.4: Match the number in the ones, tens, hundreds and thousands position to a pictorial representation or manipulative of the value.
M. 2.11.5: Model written method for recording horizontal and vertical addition and subtraction problems.
M. 2.11.6: Represent two- and three-digit numbers with multiple models.
Examples: models – base ten blocks, number lines, linking cubes, straw bundles
M. 2.11.7: Recall single-digit addition and subtraction facts.
M. 2.11.8: Add and subtract within 20, e.g., by using objects or drawings to represent the problem.

12. Mentally add 10 or 100 to a given number 100 – 900, and mentally subtract 10 or 100 from a given number 100 – 900. [2-NBT8]

Objectives:

M. 2.12.1: Demonstrate conceptual understanding of adding or subtracting 10 using concrete models.
M. 2.12.2: Recognize the place value of ones, tens and hundreds.
M. 2.12.3: Count forward and backward by 100.
M. 2.12.4: Count forward and backward by 10.
M. 2.12.5: Recall single-digit subtraction facts.
M. 2.12.6: Recall single-digit addition facts.

13. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) [2-NBT9]

Objectives:

M. 2.13.1: Explain addition and subtraction problems using concrete objects, pictures.
M. 2.13.2: Use multiple strategies to add and subtract including counting on, counting back and using doubles.
M. 2.13.3: Recall single-digit subtraction facts.
M. 2.13.4: Recall single-digit addition facts.
M. 2.13.5: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Measurement and Data

Measure and estimate lengths in standard units.

14. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. [2-MD1]

Objectives:

M. 2.14.1: Identify units of measurement for length.
Examples: inches, feet, yard; centimeter, meters
M. 2.14.2: Demonstrate how to use measurement tools.
Example: avoiding gaps and overlaps
M. 2.14.3: Identify measurement tools.
M. 2.14.4: Model measuring using non-standard units.
M. 2.14.5: Order three objects by length.
M. 2.14.6: Compare the lengths of two objects indirectly by using a third object.
M. 2.14.7: Describe measureable attributes of objects such as length or weight.

15. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. [2-MD2]

Objectives:

M. 2.15.1: Identify units of measurement for length.
Examples: inches, feet, yard; centimeter, meters
M. 2.15.2: Demonstrate how to use measurement tools.
Example: avoiding gaps and overlaps
M. 2.15.3: Identify units of measure on measurement tools.
M. 2.15.4: Use vocabulary related to comparison of length.
Examples: longer, shorter, longest, shortest, taller
M. 2.15.5: Identify numbers one to 50.

16. Estimate lengths using units of inches, feet, centimeters, and meters. [2-MD3]

Objectives:

M. 2.16.1: Define estimate.
M. 2.16.2: Measure objects using standard and non-standard units.
M. 2.16.3: Identify units of measure on measurement tools.
M. 2.16.4: Model measuring using non-standard units.
M. 2.16.5: Use vocabulary related to comparison of length.
Examples: longer, shorter, longest, shortest, and taller

17. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. [2-MD4]

Objectives:

M. 2.17.1: Measure objects using standard units.
M. 2.17.2: Record lengths with appropriate units.
M. 2.17.3: Use subtraction within 20 to solve problems.
M. 2.17.4: Compare length using non-standard units to determine which is longer.
M. 2.17.5: Use vocabulary related to comparison of length.
Examples: longer, shorter, longest, shortest, and taller

Relate addition and subtraction to length.

18. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. [2-MD5]

Objectives:

M. 2.18.1: Use addition and subtraction within 20 to solve one-step addition and subtraction word problems with an unknown number.
M. 2.18.2: Understand key words in addition and subtraction word problems involving length.
Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how many more, how many are left, in all, inches, feet, yards, longer, shorter, farther, closer
M. 2.18.3: Locate the unknown number regardless of position.
M. 2.18.4: Add and subtract within 50, e.g., by using objects or drawings to represent the problem.
M. 2.18.5: Model writing equations from word problems.
M. 2.18.6: Identify units of measurement for length.
Examples: inches, feet, yard; centimeter, meters
M. 2.18.7: Apply signs +, -, = to actions of joining and separating sets.

19. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number line diagram. [2-MD6]

Objectives:

M. 2.19.1: Recognize that each successive number name refers to a quantity that is one larger; and each previous number name refers to a quantity that is one less.
M. 2.19.2: Use a number line to add and subtract within 10.
M. 2.19.3: Write numerals 0 to 100.
M. 2.19.4: Trace numerals 0 to 100.

Work with time and money.

20. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. [2-MD7]

Objectives:

M. 2.20.1: Tell and write time in hours and half-hours using analog and digital clocks.

M. 2.20.2: Recognize vocabulary terms related to time measurements.
Examples: minute, hour, half hour, o'clock, morning, evening, a.m., p.m.

M. 2.20.3: Illustrate time to hour and half hour.
Example: Given the time 3:00, illustrate long hand and short hand positions on a clock.

M. 2.20.4: Identify the short hand as the hour hand, and the long hand as the minute hand on an analog clock.

M. 2.20.5: Identify the first number as the hour, and the numbers after the colon as the minutes on a digital clock.

M. 2.20.6: Write numerals 0 to 59.

M. 2.20.7: Recognize numerals 0 to 59.

M. 2.20.8: Count to 60 by fives.

M. 2.20.9: Distinguish between analog and digital clocks.

21. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. [2-MD8]
Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Objectives:

M. 2.21.1: Determine the monetary value of a set of like and unlike bills.

M. 2.21.2: Determine the monetary value of a set of like and unlike coins.

M. 2.21.3: Apply addition and subtraction strategies.

M. 2.21.4: Understand key words in addition and subtraction word problems involving money.
Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how much more, how much is left, in all, cents, dollar, change, paid, total

M. 2.21.5: Count forward from a given number by ones, fives, tens, and twenty-fives.

M. 2.21.6: Identify coins and bills and their value.

M. 2.21.7: Identify symbols for dollar ($), cent (¢).

M. 2.21.8: Identify coins by name including penny, nickel, dime and quarter.

M. 2.21.9: Sort pennies, nickels, dimes, and quarters.

M. 2.21.10: Count 10 objects.
Examples: pennies and dollar bills

Represent and interpret data.

22. Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by making a line plot where the horizontal scale is marked off in whole-number units. [2-MD9]

Objectives:

M. 2.22.1: Define length and line plot.
M. 2.22.2: Use vocabulary related to comparison of length.
Examples: longer, shorter, longest, shortest, taller
M. 2.22.3: Demonstrate rounding up to the nearest whole unit on measurement tools.
M. 2.22.4: Demonstrate measuring length using standard units.
M. 2.22.5: Describe a line plot.
M. 2.22.6: Model measuring length using standard units.
M. 2.22.7: Identify objects by length.
M. 2.22.8: Sort objects according to length.
M. 2.22.9: Explore objects in relationship to length.

23. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (See Appendix A, Table 1.) [2-MD10]

Objectives:

M. 2.23.1: Use addition and subtraction within 20 to solve addition and subtraction word problems with an unknown number.
M. 2.23.2: Describe picture graph and bar graph.
M. 2.23.3: Demonstrate conceptual understanding of adding or subtracting using a variety of materials.
M. 2.23.4: Use vocabulary related to comparing data.
Examples: more than, less than, most, least, equal
M. 2.23.5: Recognize attributes of data displays.
M. 2.23.6: Locate information on data displays.
M. 2.23.7: Classify objects into given categories.
M. 2.23.8: Sort the categories by count.
M. 2.23.9: Recognize different types of data displays.
M. 2.23.10: Count objects up to 50.

Geometry

Reason with shapes and their attributes.

24. Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. [2-G1]

Objectives:
M. 2.24.1: Define side, angle, face, closed, and open.
M. 2.24.2: Use vocabulary related to shape attributes.
Examples: sides, angles, face, closed, open
M. 2.24.3: Trace shapes.
M. 2.24.4: Sort triangles, quadrilaterals, pentagons, hexagons, and cubes.
M. 2.24.5: Explore triangles, quadrilaterals, pentagons, hexagons, and cubes.

25. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them. [2-G2]

Objectives:
M. 2.25.1: Define rows, columns, and total.
M. 2.25.2: Identify rectangle.
M. 2.25.3: Count to 20 by ones.
M. 2.25.4: Trace partitions in a rectangle.

26. Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds, half of, a third of,* etc.; and describe the whole as two halves, three thirds, or four fourths. Recognize that equal shares of identical wholes need not have the same shape. [2-G3]

Objectives:
M. 2.26.1: Define halves, thirds, fourths, quarters, whole, parts (shares) and equal.
M. 2.26.2: Distinguish between equal and non-equal parts.
M. 2.26.3: Model partitioning circles and rectangles.
M. 2.26.4: Decompose pictures made of simple shapes.
M. 2.26.5: Identify squares, circles, triangles and rectangles.
M. 2.26.6: Explore shapes or figures that can be decomposed into smaller equal parts.

GRADE 3

Students will:

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. [3-OA1]
   Example: Describe a context in which a total number of objects can be expressed as 5 × 7.

Objectives:

M. 3.1.1: Identify and define the parts of a multiplication problem including factors, multiplier, multiplicand and product.

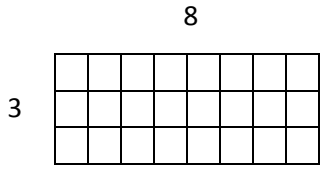
M. 3.1.2: Use multiplication to find the total number of objects arranged in rectangular arrays based on columns and rows.

M. 3.1.3: Write an equation to express the product of the multipliers (factors).

M. 3.1.4: Relate multiplication to repeated addition and skip counting.

M. 3.1.5: Apply concepts of multiplication through the use of manipulatives, number stories, skip-counting arrays, area of a rectangle, or repeated addition.

Examples: array-



Repeated addition- 8 + 8 + 8=24

M. 3.1.6: Apply basic multiplication facts through 9 x 9 using manipulatives, solving problems, and writing number stories.

M. 3.1.7: Solve addition problems with multiple addends.

M. 3.1.8: Represent addition using manipulatives.

2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. [3-OA2]
   Example: Describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Objectives:

M. 3.2.1: Identify and define the parts of a division problem including divisor, dividend, and quotient.

M. 3.2.2: Model grouping with basic division facts partitioned equally (e.g. 8 ÷ 2).

M. 3.2.3: Recognize division as either repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication.

M. 3.2.4: Apply properties of operations as strategies to subtract.

M. 3.2.5: Subtract within 20.

M. 3.2.6: Represent equal groups using manipulatives.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Appendix A, Table 2.) [3-OA3]

Objectives:

M. 3.3.1: Demonstrate computational understanding of multiplication and division by solving authentic problems with multiple representations using drawings, words, and/or numbers.

M. 3.3.2: Identify key vocabulary words to solve multiplication and division word problems.
Examples: times, every, at this rate, each, per, equal/equally, in all, total

M. 3.3.3: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

M. 3.3.4: Recall basic multiplication facts.

M. 3.3.5: Add and subtract within 20.

M. 3.3.6: Represent repeated addition, subtraction, and equal groups using manipulatives.

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. [3-OA4]
Example: Determine the unknown number that makes the equation true in each of the equations, 8 × ? = 48, 5 = □ ÷ 3, and 6 × 6 = ?.

Objectives:

M. 3.4.1: Use arrays to show equal groups in multiplication and division.

M. 3.4.2: Recall basic multiplication facts.

M. 3.4.3: Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

M. 3.4.4: Represent repeated addition, repeated subtraction, and equal groups using manipulatives.

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) [3-OA5]
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication)
3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication)
Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property)

Objectives:

M. 3.5.1: Define properties of operations.

M. 3.5.2: Apply basic multiplication facts.

M. 3.5.3: Apply properties of operations as strategies to add and subtract.

M. 3.5.4: Count to answer "how many?" questions about as many as 30 things arranged in a rectangular array.

6. Understand division as an unknown-factor problem. [3-OA6]
   Example: Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Objectives:

M. 3.6.1: Apply divisibility rules for 2, 5, and 10.
Example: Recognizing that 32 is divisible by 2 because the digit in the ones place is even.

M. 3.6.2: Apply basic multiplication facts.

M. 3.6.3: Understand subtraction as an unknown-addend problem.

M. 3.6.4: Recognize division as repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication.

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. [3-OA7]

Objectives:

M. 3.7.1: Name the first 10 multiples of each one-digit natural number.
Example: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70

M. 3.7.2: Recognize multiplication as repeated addition, and division as repeated subtraction.

M. 3.7.3: Apply properties of operations as strategies to add and subtract.

M. 3.7.4: Recall basic addition and subtraction facts.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order [Order of Operations].) [3-OA8]

Objectives:

M. 3.8.1: Define the identity property of addition and multiplication.
Examples: Addition 7 + 0 = 7, 0 + 7 = 7
Multiplication 450 x 1 = 450, 1 x 450 = 450

M. 3.8.2: Estimating sums and differences using multiple methods, including compatible numbers and rounding, to judge the reasonableness of an answer.
Examples: Compatible numbers 23 + 38 is approximately 25 + 40
Rounding 286 is approximately 300
+339 +300

M. 3.8.3: Apply commutative, associative, and identity properties for all operations to solve problems.

M. 3.8.4: Identify a rule when given a pattern.
Examples: Multiplication and division—determining from the information on the chart below a rule to be "Input x 3 = Output"

Input	2	3	5	7	12
Output	6	9	15	21	36

addition and subtraction—determining from the information on the chart below a rule to be "Input +8 = Output"

Input	1	17	21	28	30
Output	9	25	29	36	38

M. 3.8.5: Solve addition and subtraction problems, including word problems, involving one-and two-digit numbers with and without regrouping, using multiple strategies.
Example: strategies-using concrete objects, mental calculations, paper-and-pencil activities.
M. 3.8.6: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
M. 3.8.7: Represent multiplication and division with manipulatives.
M. 3.8.8: Recall basic addition and subtraction facts.

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. [3-OA9]
Example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Objectives:
M. 3.9.1: Define arithmetic patterns: geometric or numeric.
M. 3.9.2: Explain arithmetic patterns using properties of operations.
Example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed (separated into parts) into two equal addends.
M. 3.9.3: Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table).
Examples: Continue a geometric pattern O Δ □ O Δ □ __ __ __ by drawing the next three shapes.
Sample Answer: O Δ □
Complete the numerical pattern for the following chart when given the rule, "Input + 5 = Output." Sample Answer: "Input 5, Output 10;" "Input 9, Output 14."

Input	Output
3	8
4	9
5	?
7	12
9	?
12	17

M. 3.9.4: Construct repeating and growing patterns with a variety of representations.

M. 3.9.5: Demonstrate computational fluency, including quick recall, of addition and multiplication facts.
M. 3.9.6: Duplicate an existing pattern.
Example: Duplicate a numerical or geometric pattern.
M. 3.9.7: Skip count.
Example: count by twos, fives, or tens.
M. 3.9.8: Represent addition and multiplication with manipulatives.

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

10. Use place value understanding to round whole numbers to the nearest 10 or 100. [3-NBT1]

Objectives:
M. 3.10.1: Define rounding.
M. 3.10.2: Round whole numbers from 100 to 999 using whole numbers from 10 to 99.
M. 3.10.3: Model rounding whole numbers to the nearest 100.
M. 3.10.4: Round whole numbers from 10 to 99 using whole numbers from 1 to 9.
M. 3.10.5: Model rounding whole numbers to the nearest 10.
M. 3.10.6: Identify the steps in rounding two- and three-digit numbers.
Example: Identify the digit that may change and the number to the right.
M. 3.10.7: Round whole numbers from 1 to 9 and model to show proficiency.
M. 3.10.8: Understand that the two digits of a two-digit number represent amounts of tens and ones.
M. 3.10.9: Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value.

11. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. [3-NBT2]

Objectives:
M. 3.11.1: Define the commutative and associative properties of addition and subtraction.
M. 3.11.2: Subtract within 100 using strategies and algorithms based on the relationship between addition and subtraction.
M. 3.11.3: Subtract within 100 using strategies and algorithms based on properties of operations.
M. 3.11.4: Subtract within 100 using strategies and algorithms based on place value.
M. 3.11.5: Add within 100 using strategies and algorithms based on the relationship between addition and subtraction.
M. 3.11.6: Add within 100 using strategies and algorithms based on properties of operations.
M. 3.11.7: Add within 100 using strategies and algorithms based on place value.
M. 3.11.8: Recall basic addition and subtraction facts.

12. Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. [3-NBT3]

Objectives:

M. 3.12.1: Model place value by multiplying vertically.
M. 3.12.2: Model properties of operations by multiplying horizontally.
M. 3.12.3: Recall basic multiplication facts.
M. 3.12.4: Recall multiplication as repeated addition.
M. 3.12.5: Apply properties of operations as strategies to add.
Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

Number and Operations – Fractions

(Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Develop understanding of fractions as numbers.

13. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts and size $\frac{1}{b}$. [3-NF1]

Objectives:

M. 3.13.1: Define fraction, numerator, and denominator.

M. 3.13.2: Identify the parts of a fraction $\frac{a}{b}$ as the quantity formed by *a* parts and size $\frac{1}{b}$.

Example:

$\frac{2}{6}$

$\underline{a}$ = 2 parts of the fraction → numerator
b = the whole part of the fraction (6 parts)→ denominator

M. 3.13.3: Label numerator, denominator, and fraction bar.
M. 3.13.4: Identify parts of a whole with two, three, or four equal parts.
M. 3.13.5: Distinguish between equal and non-equal parts.
M. 3.13.6: Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of.

14. Understand a fraction as a number on the number line; represent fractions on a number line diagram. [3-NF2]

Objectives:
M. 3.14.1: Recognize fractions as lengths from zero to one.
M. 3.14.2: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number diagram.
M. 3.14.3: Identify a number line.

a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. [3-NF2a]

Objectives:
M. 3.14a.1: Recognize whole numbers as lengths from zero to one.
M. 3.14a.2: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number diagram.
M. 3.14a.3: Identify a number line.

b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. [3-NF2b]

Objectives:
M. 3.14b.1: Label the fractions on a pre-made number line diagram.
M. 3.14b.2: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number diagram.
M. 3.14b.3: Recognize a number line diagram with equally spaced points.

15. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. [3-NF3]

Objectives:

M. 3.15.1: Define equivalent.
M. 3.15.2: Recognize pictorial representations of equivalent fractions.
M. 3.15.3: Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler.
M. 3.15.4: Recognize that equal shares of identical wholes need not have the same shape.
M. 3.15.5: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. [3-NF3a]

Objectives:

M. 3.15a.1: Label a fraction with multiple representations.
M. 3.15a.2: Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.).
M. 3.15a.3: Partition circles and rectangles into two and four equal shares; and describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of.

b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. [3-NF3b]

Objectives:

M. 3.15b.1: Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler.
M. 3.15b.2: Label a pictorial representation.
M. 3.15b.3: Recognize that a fraction is a part of a whole.
M. 3.15b.4: Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. [3-NF3c]

Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

Objectives:

M. 3.15c.1: Define numerator and denominator.

M. 3.15c.2: Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths.

M. 3.15c.3: Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.).

M. 3.15c.4: Identify parts of a whole.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. [3-NF3d]

Objectives:

M. 3.15d.1: Represent a fraction with a pictorial model.

M. 3.15d.2: Identify <, >, and = signs.

M. 3.15d.3: Recognize that equal shares of identical wholes need not have the same shape.

M. 3.15d.4: Recognize that a whole can be partitioned into equal parts (halves, fourths, eighths, etc.).

M. 3.15d.5: Order three objects by length; compare the lengths of two objects indirectly by using a third object.

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

16. Tell and write time to the nearest minute, and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. [3-MD1]

Objectives:

M. 3.16.1: Compare equivalent units of time using hours and minutes.
Examples: 60 minutes = one hour, 30 minutes = one half of an hour

M. 3.16.2: Recognize key vocabulary and/or phrases associated with time.
Examples: Quarter 'til = 15 minutes before; half past the hour = 30 minutes after the hour

M. 3.16.3: Compare the lengths of time to complete everyday activities
Examples: Brushing your teeth = about 2 minutes; riding the bus = about 20 minutes.

M. 3.16.4: Tell and write time in hours and half-hours using analog and digital clocks.

M. 3.16.5: Recognize hour, minute, and second hands on an analog clock.

M. 3.16.6: Count by 5's to 60.

17. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems [problems involving notions of "times as much"].) (See Appendix A, Table 2.) [3-MD2]

Objectives:

M. 3.17.1: Define liquid volume, mass, grams, kilograms, and liters.
M. 3.17.2: Recognize how the standard units of measure compare to one another.
M. 3.17.3: Identify key terms for word problems.
Examples: Difference, altogether, in all, between
M. 3.17.4: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
M. 3.17.5: Recall basic addition, subtraction, multiplication, and division facts.
M. 3.17.6: Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object.

Represent and interpret data.

18. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. [3-MD3]
Example: Draw a bar graph in which each square in the bar graph might represent 5 pets.

Objectives:

M. 3.18.1: Define picture graph, bar graph, and data.
M. 3.18.2: Interpret the data to solve problems.
M. 3.18.3: Identify the parts of a graph (x-axis, y-axis, title, key, equal intervals, labels).
M. 3.18.4: Locate the data on a picture graph and a bar graph.
M. 3.18.5: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
M. 3.18.6: Directly compare two objects, with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference.

19. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters. [3-MD4]

Objectives:

M. 3.19.1: Define line plot.
M. 3.19.2: Identify the parts of a line plot.
M. 3.19.3: Measure objects to the nearest inch.
M. 3.19.4: Identify one-inch units on a ruler starting with 0.
M. 3.19.5: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
M. 3.19.6: Directly compare two objects, with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

20. Recognize area as an attribute of plane figures, and understand concepts of area measurement. [3-MD5]

Objectives:
M. 3.20.1: Define plane figures.
M. 3.20.2: Differentiate between closed and open figures.
M. 3.20.3: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
M. 3.20.4: Identify shapes as two-dimensional (i.e., lying in a plane, "flat").
M. 3.20.5: Correctly name shapes regardless of their orientations or overall size.

a. A square with side length 1 unit called "a unit square," is said to have "one square unit" of area and can be used to measure area. [3-MD5a]

Objectives:
M. 3.20a.1: Define length.
M. 3.20a.2: Recognize that units of measure must be equal.
M. 3.20a.3: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.
M. 3.20a.4: Recognize that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. [3-MD5b]

Objectives:
M. 3.20b.1: Define area.
M. 3.20b.2: Recognize that "n square units" is a variable.
M. 3.20b.3: Recognize that unit squares are equal.
M. 3.20b.4: Identify shapes as two-dimensional (i.e., lying in a plane, "flat").

21. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). [3-MD6]

Objectives:
M. 3.21.1: Recognize that unit squares are equal.
M. 3.21.2: Define the units of measurement (cm, m, in, ft).
M. 3.21.3: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

22. Relate area to the operations of multiplication and addition. [3-MD7]

Objectives:
M. 3.22.1: Recognize arrays as multiplication or repeated addition.
M. 3.22.2: Recall basic addition and multiplication facts.
M. 3.22.3: Build and draw shapes to possess defining attributes.
M. 3.22.4: Compose simple shapes to form larger shapes.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. [3-MD7a]

Objectives:
M. 3.22a.1: Recognize arrays as multiplication or repeated addition.
M. 3.22a.2: Identify units of measure as equal units.
M. 3.22a.3: Build and draw shapes to possess defining attributes.
M. 3.22a.4: Compose simple shapes to form larger shapes.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. [3-MD7b]

Objectives:
M. 3.22b.1: Recall basic multiplication facts.
M. 3.22b.2: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
M. 3.22b.3: Recognize multiplication as repeated addition.
M. 3.22b.4: Add within 100.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. [3-MD7c]

Objectives:
M. 3.22c.1: Define distributive property.
M. 3.22c.2: Label pre-made arrays.
M. 3.22c.3: Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
M. 3.22c.4: Add within 100.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into nonoverlapping rectangles and adding the areas of the nonoverlapping parts, applying this technique to solve real-world problems. [3-MD7d]

Objectives:
M. 3.22d.1: Label pre-made arrays.
M. 3.22d.2: Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
M. 3.22d.3: Recall basic addition and multiplication facts.
M. 3.22d.4: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles).
M. 3.22d.5: Identify a rectangle.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

23. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. [3-MD8]

Objectives:
M. 3.23.1: Define perimeter.
M. 3.23.2: Recall the formula for perimeter (P= L+L+W+W or P=2L + 2W)
M. 3.23.3: Recall basic addition and multiplication facts.
M. 3.23.4: Build and draw shapes to possess defining attributes.
M. 3.23.5: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
M. 3.23.6: Describe measurable attributes of objects such as length or weight.

Geometry

Reason with shapes and their attributes.

24. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. [3-G1]

Objectives:
M. 3.24.1: Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.).
M. 3.24.2: Recognize and draw shapes having specified attributes such as a given number of angles.
M. 3.24.3: Build and draw shapes to possess defining attributes.
M. 3.24.4: Sort shapes into categories.

25. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. [3-G2]

    Example: Partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Objectives:

M. 3.25.1: Recognize a fraction as part of a whole.

M. 3.25.2: Decompose a large pre-made shape using smaller shapes.

M. 3.25.3: Compose a large pre-made shape using smaller shapes.

M. 3.25.4: Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.

M. 3.25.5: Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths.

GRADE 4

Students will:

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. [4-OA1]

Objectives:
M. 4.1.1: Use arrays to show equal groups in multiplication.
M. 4.1.2: Recall basic multiplication facts.
M. 4.1.3: Interpret the products of whole numbers.
M. 4.1.4: Demonstrate computational fluency, including quick recall of addition and subtraction facts.
M. 4.1.5: Recognize multiplication as repeated addition.

2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Appendix A, Table 2.) [4-OA2]

Objectives:
M. 4.2.1: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
M. 4.2.2: Recognize key terms to solve word problems.
Examples: in all, how much, how many, in each
M. 4.2.3: Apply properties of operations as strategies to add.
M. 4.2.4: Recall basic multiplication facts.
M. 4.2.5: Demonstrate computational fluency, including quick recall of addition and subtraction facts.

3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. [4-OA3]

Objectives:
M. 4.3.1: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
M. 4.3.2: Solve single-step word problems.
M. 4.3.3: Recognize key terms to solve word problems.
Examples: in all, how much, how many, in each
M. 4.3.4: Solve division problems without remainders.
M. 4.3.5: Recall basic addition, subtraction, and multiplication facts.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. [4-OA4]

Objectives:
M. 4.4.1: Define factors, prime number, and composite number.
M. 4.4.2: Apply properties of operations as strategies to multiply and divide.
M. 4.4.3: Identify all factor pairs for a whole number in the range 1-20.
M. 4.4.4: Name the first ten multiples of each one-digit natural number.
M. 4.4.5: Recall basic multiplication facts.
M. 4.4.6: Count within 1000; skip-count by 5s, 10s, and 100s.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. [4-OA5]
   Example: Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence, and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Objectives:
M. 4.5.1: Identify arithmetic patterns, including patterns in the addition table or multiplication table; and explain them using properties of operations
M. 4.5.2: Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table).
Examples:
Continue a geometric pattern O Δ □ O Δ □ __ __ __ by drawing the next three shapes.
Sample Answer: O Δ □

Complete the numerical pattern for the following chart when given the rule, "Input + 5 = Output."
Sample Answer: "Input 5, Output 10;" "Input 9, Output 14."

Input	Output
3	8
4	9
5	?
7	12
9	?
12	17

M. 4.5.3: Construct repeating and growing patterns with a variety of representations.
M. 4.5.4: Continue an existing pattern.
M. 4.5.5: Identify arithmetic patterns.
M. 4.5.6: Demonstrate computational fluency, including quick recall, of addition multiplication facts.

Number and Operations in Base Ten

(Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)

Generalize place value understanding for multi-digit whole numbers.

6. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. [4-NBT1]
   Example: Recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

Objectives:

M. 4.6.1: Use place value understanding to round whole numbers to the nearest 10 or 100.
M. 4.6.2: Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
M. 4.6.3: Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
M. 4.6.4: Recall basic multiplication facts.
M. 4.6.5: Recall that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
M. 4.6.6: Recognize that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones)
M. 4.6.7: Recognize that 100 can be thought of as a bundle of ten tens, called a "hundred."

7. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. [4-NBT2]

Objectives:

M. 4.7.1: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons.
M. 4.7.2: Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
M. 4.7.3: Convert a number written in expanded notation to standard form.

8. Use place value understanding to round multi-digit whole numbers to any place. [4-NBT3]

Objectives:

M. 4.8.1: Use place value understanding to round whole numbers to the nearest 10 or 100.
M. 4.8.2: Model rounding whole numbers to the nearest 100.
M. 4.8.3: Round whole numbers from 100 to 999 using whole numbers from 10 to 99.
M. 4.8.4: Model rounding whole numbers to the nearest 10.
M. 4.8.5: Round whole numbers from 10 to 99 using whole numbers from 1 to 9.
M. 4.8.6: Round whole numbers from 1 to 9 and model to show proficiency.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

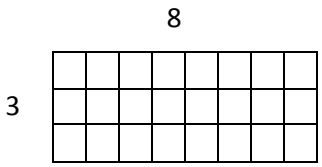
9. Fluently add and subtract multi-digit whole numbers using the standard algorithm. [4-NBT4]

Objectives:
M. 4.9.1: Add and subtract within 1000.
M. 4.9.2: Apply signs +, -, and = to actions of joining and separating sets.
M. 4.9.3: Add and subtract single-digit numbers.
M. 4.9.4: Recall basic addition and subtraction facts.

10. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [4-NBT5]

Objectives:
M. 4.10.1: Multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.
M. 4.10.2: Multiply single-digit numbers.
M. 4.10.3: Recall basic multiplication facts.
M. 4.10.4: Apply concepts of multiplication through the use of manipulatives, number stories, skip-counting arrays, area of a rectangle, or repeated addition.
Examples:

Array-



Repeated addition- 8 + 8 + 8=24

11. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [4-NBT6]

Objectives:
M. 4.11.1: Divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8).
M. 4.11.2: Divide within 100, using strategies such as properties of operations.
M. 4.11.3: Multiply within 100, using strategies such as properties of operations.
M. 4.11.4: Multiply within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that 8 x 5 = 4=, one knows 40 ÷ 5 = 8).
M. 4.11.5: Recall products of two one-digit numbers.
M. 4.11.6: Name the first 10 multiples of each one-digit natural number.
Example: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
M. 4.11.7: Recall basic addition, subtraction, and multiplication facts.

Number and Operations – Fractions

(Grade 4 expectations in this domain are limited to fractions with denominations 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

Extend understanding of fraction equivalence and ordering.

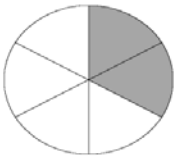
12. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(nxa)}{(nxb)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [4-NF1]

Objectives:

M. 4.12.1: Define fraction, numerator and denominator.

M. 4.12.2: Recognize fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts.

Example:



$\frac{2}{6}$

M. 4.12.3: Identify the parts of a fraction $\frac{a}{b}$ as the quantity formed by a parts and size $\frac{1}{b}$.

In the example above, a = 2 parts of the fraction → numerator
b = the whole part of the fraction (6 parts) → denominator

M. 4.12.4: Recognize fractions as numerals that may represent division problems.

M. 4.12.5: Label numerator, denominator, and fraction bar.

M. 4.12.6: Identify parts of a whole with two, three, or four equal parts.

M. 4.12.7: Recognize that equal shares of identical wholes need not have the same shape.

M. 4.12.8: Distinguish between equal and non-equal parts.

13. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. [4-NF2]

Objectives:

M. 4.13.1: Identify fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts and size $\frac{1}{b}$.

M. 4.13.2: Identify a fraction as a number on the number line; represent fractions on a number line diagram.

M. 4.13.3: Recognize a fraction as a number on the number line.

M. 4.13.4: Represent fractions on a number line diagram.

M. 4.13.5: Recognize fractions as numerals that may represent division problems.

M. 4.13.6: Label numerator, denominator, and fraction bar.
M. 4.13.7: Identify parts of a whole with two, three, or four equal parts.
M. 4.13.8: Distinguish between equal and non-equal parts.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

14. Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. [4-NF3]

Objectives:
M. 4.14.1: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
M. 4.14.2: Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line.
M. 4.14.3: Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
M. 4.14.4: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

M. 4.14.5: Label a fraction with multiple representations.
M. 4.14.6: Recognize pictorial representations of equivalent fractions.
M. 4.14.7: Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler.
M. 4.14.8: Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.).

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. [4-NF3a]

Objectives:
M. 4.14a.1: Identify numerator and denominator.
M. 4.14a.2: Recall basic addition and subtraction facts.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. [4-NF3b]

Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

Objectives:
M. 4.14b.1: Demonstrate an understanding of fractional parts.
M. 4.14b.2: Recall basic addition and subtraction facts.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. [4-NF3c]

Objectives:
M. 4.14c.1: Define mixed numbers.
M. 4.14c.2: Recall basic addition and subtraction facts.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. [4-NF3d]

Objectives:
M. 4.14d.1: Demonstrate an understanding of fractional parts.
M. 4.14d.2: Solve basic word problems using whole numbers.
M. 4.14d.3: Express parts of a whole as a fraction.
M. 4.14d.4: Write number sentences for word problems.
M. 4.14d.5: Identify key terms in word problems.
M. 4.14d.6: Recall basic addition and subtraction facts.

15. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. [4-NF4]

Objectives:
M. 4.15.1: Recognize fractions in their simplest forms.
M. 4.15.2: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
M. 4.15.3: Demonstrate an understanding of fractional parts.
M. 4.15.4: Apply properties of operations as strategies to multiply and divide.
M. 4.15.5: Recall basic multiplication facts.

a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. [4-NF4a]

Example: Use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.

Objectives:
M. 4.15a.1: Define multiple.
M. 4.15a.2: Compare two fractions with the same numerator or the same denominator by reasoning about their size.
M. 4.15a.3: Recognize that comparisons are valid only when the two fractions refer to the same whole.
M. 4.15a.4: Record results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
M. 4.15a.5: Name the first ten multiples of each one-digit natural number.
M. 4.15a.6: Recall basic multiplication facts.

b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. [4-NF4b]

Example: Use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, $n \times (\frac{a}{b}) = \frac{(nxa)}{b}$.)

Objectives:

M. 4.15b.1: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

M. 4.15b.2: Solve simple fractions using multiplication strategies.

M. 4.15b.3: Recognize equivalent forms of fractions.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. [4-NF4c]

Example: If each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between which two whole numbers does your answer lie?

Objectives:

M. 4.15c.1: Multiply proper fractions with common denominators 2-10.

M. 4.15c.2: Solve word problems using whole numbers.

M. 4.15c.3: Write number sentences for word problems.

M. 4.15c.4: Identify key terms in word problems.

M. 4.15c.5: Multiply and divide within 100.

Understand decimal notation for fractions, and compare decimal fractions.

16. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) [4-NF5]

    Example: Express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

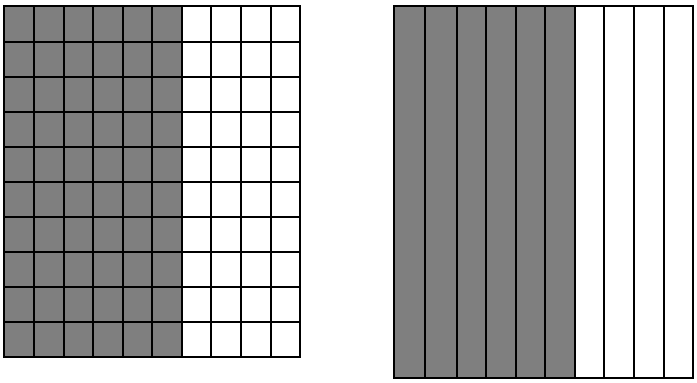
Objectives:

M. 4.16.1: Recognize equivalent forms of fractions and decimals.

M. 4.16.2: Demonstrate equivalent fractions using concrete objects or pictorial representation.

M. 4.16.3: Recognize pictorial representations of equivalent fractions and decimals in tenths and hundredths.

Example:



0.60 = 0.6

M. 4.16.4: Identify place value of decimals to the tenths and hundredths.

M. 4.16.5: Use place value understanding to round whole numbers to the nearest 10 or 100.

17. Use decimal notation for fractions with denominators 10 or 100. [4-NF6]

    Example: Rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Objectives:

M. 4.17.1: Define tenths, hundredths, decimal notation.

M. 4.17.2: Recognize equivalent forms of fractions and decimals.

M. 4.17.3: Recognize that endpoints locate $\frac{a}{b}$ on a number line.

M. 4.17.4: Identify place value of decimals to the tenths and hundredths.

M. 4.17.5: Label fraction parts.

Examples: numerator, denominator, fraction bar

M. 4.17.6: Use place value understanding to round whole numbers to the nearest 10 or 100.

18. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. [4-NF7]

Objectives:

M. 4.18.1: Compare two fractions with the same numerator or the same denominator by reasoning about their size.
M. 4.18.2: Recognize that comparisons are valid only when the two fractions refer to the same whole.
M. 4.18.3: Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
M. 4.18.4: Convert fractions to decimals.
M. 4.18.5: Compare two decimals to tenths.
M. 4.18.6: Compare whole numbers.
M. 4.18.7: Identify comparison symbols.
Examples: >, <, and =

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

19. Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. [4-MD1]
Examples: Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)

Objectives:

M. 4.19.1: Define conversion.
M. 4.19.2: Define length, kilometers, meters and centimeters.
M. 4.19.3: Define weight, kilograms, grams, pounds, ounces, liters and milliliters.
M. 4.19.4: Define hour, minute, second.
M. 4.19.5: Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters.
M. 4.19.6: Identify standard units of measurement equivalents.
Examples: 60 minutes equals 1 hour, 16 ounces equals 1 pound
M. 4.19.7: Match measurement units to abbreviations.
Examples: kilometers (km), meters (m), centimeters (cm), kilograms (kg), grams (g), pounds (lb), ounces (oz), liters (l), milliliters (ml)

20. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. [4-MD2]

Objectives:

M. 4.20.1: Define distance, time, elapsed time, volume, mass.
M. 4.20.2: Determine elapsed time to the day with calendars and to the hour with a clock.
M. 4.20.3: Express liquid volumes and masses of objects using standard units of grams, kilograms, and liters.
M. 4.20.4: Use addition, subtraction, multiplication and division to solve one- and two-step word problems.
M. 4.20.5: Recognize key terms to solve word problems.
M. 4.20.6: Recall basic facts for addition, subtraction, multiplication, and division.
M. 4.20.7: Identify monetary equivalents.
Examples: four quarters equal one dollar, five one-dollar bills equals five dollars

21. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. [4-MD3]
Example: Find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.

Objectives:

M. 4.21.1: Recall the formula for area (L X W).
M. 4.21.2: Recognize that unit squares are equal.
M. 4.21.3: Recall the formula for perimeter (P= L+L+W+W or P=2L + 2W).
M. 4.21.4: Recall basic addition and multiplication facts.

Represent and interpret data.

22. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. [4-MD4]
Example: From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Objectives:

M. 4.22.1: Display data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.
M. 4.22.2: Interpret data using graphs including bar, line, and circle graphs, and Venn diagrams.
M. 4.22.3: Identify the parts of a line plot.
M. 4.22.4: Recognize a line plot.
M. 4.22.5: Draw a scaled picture graph and a scaled bar graph to represent a data set.

Geometric measurement: understand concepts of angle and measure angles.

23. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. [4-MD5]

Objectives:

M. 4.23.1: Define degree, angle, ray, and vertices.

M. 4.23.2: Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces.

M. 4.23.3: Estimate angle measures using 45°, 90°, 180°, 270°, or 360°.

M. 4.23.4: Identify angle, ray, and vertices.

M. 4.23.5: Draw shapes to possess defining attributes.

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle" and can be used to measure angles. [4-MD5a]

Objectives:

M. 4.23a.1: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Example: Partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

M. 4.23a.2: Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds, half of, a third of,* etc.; and describe the whole as two halves, three thirds, or four fourths.

M. 4.23a.3: Recognize that equal shares of identical wholes need not have the same shape.

M. 4.23a.4: Demonstrate equivalent fractions using concrete objects or pictorial representations. Examples:

$\frac{2}{6}$ = $\frac{1}{3}$

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. [4-MD5b]

Objectives:
M. 4.23b.1: Define center, radius, and diameter of a circle.
M. 4.23b.2: Identify real-world examples of radius and diameter.
Examples: bicycle wheel, pizza, pie
M. 4.23b.3: Identify intervals of 1° between 0 and 5 on a protractor.
M. 4.23b.4: Skip count by fives and tens on a protractor.

24. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. [4-MD6]

Objectives:
M. 4.24.1: Define symmetry.
M. 4.24.2: Model using a protractor to draw angles.
M. 4.24.3: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
M. 4.24.4: Measure the length of an object by selecting and using appropriate tools such as a ruler.
M. 4.24.5: Measure length using standard and non-standard units of measurement.
M. 4.24.6: Plot points on grids, graphs, and maps using coordinates.
M. 4.24.7: Draw points, lines, line segments, and parallel and perpendicular lines, angles, and rays.
M. 4.24.8: Identify lines of symmetry on one-dimensional figures.

25. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world or mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. [4-MD7]

Objectives:
M. 4.25.1: Identify straight angles.
M. 4.25.2: Recognize angle measures such as 45°, 90°, 180°, 270°, 300°.
M. 4.25.3: Recall basic addition and subtraction facts.
M. 4.25.4: Skip count by fives and tens.

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

26. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. [4-G1]

Objectives:
M. 4.26.1: Define points, lines, line segments, rays, right angle, acute angle, obtuse angle, perpendicular lines, and parallel lines.
M. 4.26.2: Define two-dimensional figure.
M. 4.26.3: Recognize one-dimensional points, lines, and line segments.
M. 4.26.4: Model shapes in the world by building shapes from components.

27. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. [4-G2]

Objectives:

M. 4.27.1: Define right angle.

M. 4.27.2: Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).

M. 4.27.3: Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

M. 4.27.4: Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces.

M. 4.27.5: Identify triangles.

28. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. [4-G3]

Objectives:

M. 4.28.1: Identify line symmetric figures.

M. 4.28.2: Draw lines of symmetry on a one-dimensional figure.

M. 4.28.3: Recognize lines of symmetry on a one-dimensional figure.

GRADE 5

Students will:

Operations and Algebraic Thinking

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. [5-OA1]

Objectives:
M. 5.1.1: Define parentheses, braces, and brackets.
M. 5.1.2: Define numerical expression.
M. 5.1.3: Recognize expressions.
M. 5.1.4: Apply properties of operations as strategies to add and subtract.
M. 5.1.5: Recall properties of operations as strategies to add and subtract.
M. 5.1.6: Represent addition and subtraction with objects, mental images, drawings, expressions, or equations.

2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. [5-OA2]
   Examples: Express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18{,}932 + 921)$ is three times as large as $18{,}932 + 921$, without having to calculate the indicated sum or product.

Objectives:
M. 5.2.1: Define simple expression.
M. 5.2.2: Recall simple equations.
M. 5.2.3: Recognize properties of addition and multiplication.
M. 5.2.4: Recall addition, subtraction, multiplication, division symbols.

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. [5-OA3]
   Example: Given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Objectives:
M. 5.3.1: Construct repeating and growing patterns with a variety of representations.
M. 5.3.2: Continue an existing pattern.
M. 5.3.3: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations

M. 5.3.4: Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table).

Examples: Continue a geometric pattern O Δ □ O Δ □ __ __ __ by drawing the next three shapes.

Sample Answer: O Δ □

Complete the numerical pattern for the following chart when given the rule, "Input + 5 = Output."

Sample Answer: "Input 5 , Output 10;" "Input 9, Output 14."

Input	Output
3	8
4	9
5	?
7	12
9	?
12	17

M. 5.3.5: Recall basic addition facts.

Number and Operations in Base Ten

Understand the place value system.

4. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. [5-NBT1]

Objectives:

M. 5.4.1: Use place value understanding to round whole numbers to the nearest 10 or 100.

M. 5.4.2: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons.

M. 5.4.3: Identify that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

5. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. [5-NBT2]

Objectives:

M. 5.5.1: Define product, power of 10.

M. 5.5.2: Recognize decimal place value using visual representations.

M. 5.5.3: Recall multiplication and division facts of 10.

M. 5.5.4: Skip count forward and backward by 10.

6. Read, write, and compare decimals to thousandths. [5-NBT3]

Objectives:
M. 5.6.1: Recognize decimals as parts of a whole.
M. 5.6.2: Compare whole numbers.
M. 5.6.3: Write whole numbers in words and expanded form.
M. 5.6.4: Read whole numbers.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$. [5-NBT3a]

Objectives:
M. 5.6a.1: Define expanded notation and standard form.
M. 5.6a.2: Convert a number written in expanded to standard form.
M. 5.6a.3: Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. [5-NBT3b]

Objectives:
M. 5.6b.1: Define hundredths and thousandths.
M. 5.6b.2: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons.
M. 5.6b.3: Identify comparison symbols.
Examples: >, =, and <

7. Use place value understanding to round decimals to any place. [5-NBT4]

Objectives:
M. 5.7.1: Round multi-digit whole numbers to any place.
M. 5.7.2: Round whole numbers to the nearest 10 or 100.
M. 5.7.3: Recognize that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

8. Fluently multiply multi-digit whole numbers using the standard algorithm. [5-NBT5]

Objectives:
M. 5.8.1: Demonstrate steps in setting up a long multiplication problem.
M. 5.8.2: Multiply 2-digit numbers by 1-digit multiplier.
M. 5.8.3: Multiply 1-digit numbers by 1-digit multiplier.
M. 5.8.4: Recall basic multiplication facts.
M. 5.8.5: Recall repeated addition facts.

9. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [5-NBT6]

Objectives:

M. 5.9.1: Contrast a division equation with an example of the division algorithm.
M. 5.9.2: Illustrate the division algorithm using a one-digit divisor and a 2-digit dividend.
M. 5.9.3: Recall the properties of operations in division.
M. 5.9.4: Identify the place value of a division problem.
M. 5.9.5: Restate the inverse process of division as multiplication.
M. 5.9.6: Demonstrate the relationship between an array and multiplication.
M. 5.9.7: Recall basic multiplication facts.

10. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used. [5-NBT7]

Objectives:

M. 5.10.1: Use decimal notation for fractions with denominators 10 or 100.

Example: Rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

M. 5.10.2: Multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations.
M. 5.10.3: Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
M. 5.10.4: Apply properties of operations as strategies to multiply and divide.
M. 5.10.5: Explain why addition and subtraction strategies work, using place value and the properties of operations.
M. 5.10.6: Identify that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
M. 5.10.7: Identify that 100 can be thought of as a bundle of ten tens, called a "hundred."
M. 5.10.8: Identify that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
M. 5.10.9: Recall basic addition, subtraction, multiplication, and division facts.

Number and Operations – Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

11. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. [5-NF1]

    Example: $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)

Objectives:

M. 5.11.1: Recall a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$.

M. 5.11.2: Recall addition and subtraction of fractions as joining and separating parts referring to the same whole.

M. 5.11.3: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

M. 5.11.4: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

M. 5.11.5: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

M. 5.11.6: Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line.

M. 5.11.7: Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

M. 5.11.8: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

M. 5.11.9: Compare two fractions with the same numerator or the same denominator by reasoning about their size.

M. 5.11.10: Recall basic addition, subtraction, multiplication, and division facts.

12. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally, and assess the reasonableness of answers. [5-NF2]

    Example: Recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$ by observing that $\frac{3}{7} < \frac{1}{2}$.

Objectives:

M. 5.12.1: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

M. 5.12.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$.

M. 5.12.3: Recognize that comparisons are valid only when the two fractions refer to the same whole.

M. 5.12.4: Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

M. 5.12.5: Compare two fractions with the same numerator or the same denominator by reasoning about their size.

M. 5.12.6: Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

M. 5.12.7: Recognize a fraction as a number on the number line; represent fractions on a number line diagram.

M. 5.12.8: Recognize key terms to solve word problems.

M. 5.12.9: Apply properties of operations for addition and subtraction.

M. 5.12.10: Recall basic addition and subtraction facts.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

13. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. [5-NF3]

    Examples: Interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between which two whole numbers does your answer lie?

Objectives:

M. 5.13.1: Define mixed number.

M. 5.13.2: Recognize a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$.

M. 5.13.3: Identify that fraction $\frac{a}{b}$ is equivalent to fraction $\frac{(nxa)}{(nxb)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

M. 5.13.4: Generate equivalent fractions.

M. 5.13.5: Recognize a fraction as a number on the number line; represent fractions on a number line diagram.

M. 5.13.6: Identify fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts and size $\frac{1}{b}$.

14. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. [5-NF4]

Objectives:

M. 5.14.1: Define proper fraction.

M. 5.14.2: Multiply fractions using denominators between 2 and 5.

M. 5.14.3: Identify proper and improper fractions.

M. 5.14.4: Recall basic multiplication facts.

a. Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. [5-NF4a]

Example: Use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)

Objectives:
M. 5.14a.1: Model changing a whole number to a fraction.
M. 5.14a.2: Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
M. 5.14a.3: Label the numerator and denominator of a fraction.

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. [5-NF4b]

Objectives:
M. 5.14b.1: Count the area squares for the length and width.
M. 5.14b.2: Identify the width and length of a rectangle.

15. Interpret multiplication as scaling (resizing), by: [5-NF5]

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. [5-NF5a]
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case), explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number, and relating the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1. [5-NF5b]

Objectives:
M. 5.15.1: Define scaling.
M. 5.15.2: Define principle of fraction equivalence.
M. 5.15.3: Multiply a fraction by a whole number.
M. 5.15.4: Compare two fractions with the same numerator or the same denominator by reasoning about their size.
M. 5.15.5: Recognize that comparisons are valid only when the two fractions refer to the same whole.
M. 5.15.6: Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
M. 5.15.7: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
M. 5.15.8: Identify factor and product.
M. 5.15.9: Use comparison symbols.
Examples: >, =, or <

16. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. [5-NF6]

Objectives:

M. 5.16.1: Define improper fraction, mixed number, fraction, equations, numerator, denominator.
M. 5.16.2: Multiply proper fractions with common denominators 2-10.
M. 5.16.3: Solve problems using whole numbers.
M. 5.16.4: Write number sentences for word problems.
M. 5.16.5: Identify key terms to solve multiplication word problems.
Examples: times, every, at this rate, each, per, equal/equally, in all, total
M. 5.16.6: Recall basic multiplication facts.

17. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. However, division of a fraction by a fraction is not a requirement at this grade.) [5-NF7]

Objectives:

M. 5.17.1: Define quotient.
M. 5.17.2: Multiply a fraction by a whole number.
M. 5.17.3: Recognize key terms to solve word problems.
Examples: times, every, at this rate, each, per, equal/equally, in all, total
M. 5.17.4: Recall basic multiplication and division facts.

a. Interpret division of a unit fraction by a nonzero whole number, and compute such quotients. [5-NF7a]

Example: Create a story context for $(\frac{1}{3}) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = \frac{1}{12}$ because $(\frac{1}{12}) \times 4 = \frac{1}{3}$.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. [5-NF7b]

Example: Create a story context for $4 \div (\frac{1}{5})$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$ because $20 \times (\frac{1}{5}) = 4$.

Objectives:

M. 5.17ab.1: Recognize a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.
M. 5.17ab.2: Recognize fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.
M. 5.17ab.3: Express whole numbers as fractions.
M. 5.17ab.4: Recognize fractions that are equivalent to whole numbers.
M. 5.17ab.5: Recall basic multiplication and division facts.

c. Solve real-world problems involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. [5-NF7c]

Examples: How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

Objectives:

M. 5.17c.1: Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Example: If each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between which two whole numbers does your answer lie?

M. 5.17c.2: Recognize key terms to solve word problems.
Examples: times, every, at this rate, each, per, equal/equally, in all, total

M. 5.17c.3: Recall basic multiplication and division facts.

Measurement and Data

Convert like measurement units within a given measurement system.

18. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multistep, real-world problems. [5-MD1]

Objectives:

M. 5.18.1: Identify relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.

M. 5.18.2: Express measurements in a larger unit in terms of a smaller unit.

M. 5.18.3: Solve two-step word problems.

M. 5.18.4: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

M. 5.18.5: Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

M. 5.18.6: Recall basic addition, subtraction, multiplication, and division facts.

Represent and interpret data.

19. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$).

    Use operations on fractions for this grade to solve problems involving information presented in line plots. [5-MD2]

    Example: Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Objectives:

M. 5.19.1: Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$).

M. 5.19.2: Solve problems involving addition and subtraction of fractions by using information presented in line plots.

M. 5.19.3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.

M. 5.19.4: Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

M. 5.19.5: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.

M. 5.19.6: Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

20. Recognize volume as an attribute of solid figures, and understand concepts of volume measurement. [5-MD3]

    a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. [5-MD3a]
    b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. [5-MD3b]

Objectives:

M. 5.20ab.1: Define volume including the formulas V = l x w x h, and V = B x h.

M. 5.20ab.2: Define solid figures.

M. 5.20ab.3: Define unit cube.

M. 5.20ab.4: Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).

M. 5.20ab.5: Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

M. 5.20ab.6: Describe attributes of three-dimensional figures.

M. 5.20ab.7: Describe attributes of two-dimensional figures.

M. 5.20ab.8: Compare the unit size of volume/capacity in the metric system including milliliters and liters.

21. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. [5-MD4]

Objectives:

M. 5.21.1: Define cubic inches, cubic centimeters, and cubic feet.
M. 5.21.2: Compare the unit size of volume/capacity in the customary system including fluid ounces, cups, pints, quarts, gallons.
M. 5.21.3: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
M. 5.21.4: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
M. 5.21.5: Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
M. 5.21.6: Recall basic multiplication facts.
M. 5.21.7: Fluently add.

22. Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume. [5-MD5]

Objectives:

M. 5.22.1: Define volume.
M. 5.22.2: Recognize angle measure as additive.
M. 5.22.3: Solve addition and subtraction problems to find unknown angles on a diagram in real-world or mathematical problems, e.g., by using an equation with a symbol for the unknown angle.
M. 5.22.4: Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.
M. 5.22.5: Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
M. 5.22.6: Recognize the formula for volume.
M. 5.22.7: Recall the attributes of three-dimensional solids.
M. 5.22.8: Recall basic multiplication facts.
M. 5.22.9: Fluently add.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. [5-MD5a]

Objectives:

M. 5.22a.1: Compare the unit size of volume/capacity in the metric system including milliliters and liters.
M. 5.22a.2: Measure and estimate liquid volumes.
M. 5.22a.3: Recall basic multiplication facts.

b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. [5-MD5b]

Objectives:
M. 5.22b.1: Compare the unit size of volume/capacity in the metric system including milliliters and liters.
M. 5.22b.2: Recognize the formula for volume.
M. 5.22b.3: Recall basic multiplication facts.

c. Recognize volume as additive. Find volumes of solid figures composed of two nonoverlapping right rectangular prisms by adding the volumes of the nonoverlapping parts, applying this technique to solve real-world problems. [5-MD5c]

Objectives:
M. 5.22c.1: Describe attributes of three-dimensional figures.
M. 5.22c.2: Describe attributes of two-dimensional figures.
M. 5.22c.3: Identify solid figures.

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

23. Use a pair of perpendicular number lines, called axes, to define a coordinate system with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). [5-G1]

Objectives:
M. 5.23.1: Define ordered pair of numbers.
M. 5.23.2: Define x-axis, y-axis, and zero on a coordinate.
M. 5.23.3: Specify locations on the coordinate system.
M. 5.23.4: Illustrate vertical and horizontal number lines.
M. 5.23.5: Label x- and y-axis and zero on a coordinate.
M. 5.23.6: Locate negative numbers on a horizontal number line.
M. 5.23.7: Locate positive numbers on a horizontal number line.
M. 5.23.8: Locate positive numbers on a vertical number line.
Examples: thermometer, map

24. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. [5-G2]

Objectives:

M. 5.24.1: Define ordered pair of numbers, quadrant one, coordinate plane, and plot points.
M. 5.24.2: Label the horizontal axis (x).
M. 5.24.3: Label the vertical axis (y).
M. 5.24.4: Identify the x and y values in ordered pairs.
M. 5.24.5: Model writing ordered pairs.

Classify two-dimensional figures into categories based on their properties.

25. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. [5-G3]
    Example: All rectangles have four right angles, and squares are rectangles, so all squares have four right angles.

Objectives:

M. 5.25.1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
M. 5.25.2: Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
M. 5.25.3: Recognize attributes of shapes.
M. 5.25.4: Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.).
M. 5.25.5: Sort shapes into categories.

26. Classify two-dimensional figures in a hierarchy based on properties. [5-G4]

Objectives:

M. 5.26.1: Define vertex/vertices and angle.
M. 5.26.2: Identify that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
M. 5.26.3: Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces.
M. 5.26.4: Identify triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons based on the number of sides, angles, and vertices.

GRADE 6

Students will:

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities. [6-RP1]
   Examples: "The ratio of wings to beaks in the bird house at the zoo was 2:1 because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

Objectives:
M. 6.1.1: Define quantity, fraction, and ratio.
M. 6.1.2: Reinterpret a fraction as a ratio.
Example: Read 2/3 as 2 out of 3.
M. 6.1.3: Write a ratio as a fraction.
M. 6.1.4: Draw a model of a given fraction.
M. 6.1.5: Identify the numerator and denominator of a fraction.

2. Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. [6-RP2]
   Examples: "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)

Objectives:
M. 6.2.1: Define unit rate, proportion, and rate.
M. 6.2.2: Create a ratio or proportion from a given word problem.
M. 6.2.3: Calculate unit rate by using ratios or proportions.
M. 6.2.4: Interpret a fraction as division of the numerator by the denominator.
Example: (a/b=a÷b).
M. 6.2.5: Write a ratio as a fraction.

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. [6-RP3]

Objectives:
M. 6.3.1: Create a ratio or proportion from a given word problem, diagram, table, or equation.
M. 6.3.2: Calculate unit rate or rate by using ratios or proportions.
M. 6.3.3: Restate real world problems or mathematical problems.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. [6-RP3a]

Objectives:
M. 6.3a.1: Define coordinate plane, equivalent, input, output, and ordered pairs**.**
M. 6.3a.2: Construct a graph from a set of ordered pairs given in the table of equivalent ratios.
M. 6.3a.3: Calculate missing input and/or output values in a table.
M. 6.3a.4: Draw and label a table of equivalent ratios from given information.
M. 6.3a.5: Identify the parts of a table of equivalent ratios (input, output, etc).

b. Solve unit rate problems including those involving unit pricing and constant speed. [6-RP3b]
Example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Objectives:
M. 6.3b.1: Compute the unit rate, unit price, and constant speed.
M. 6.3b.2: Create a proportion or ratio from a given word problem.
M. 6.3b.3: Identify the two units being compared.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity measure of centers $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent. [6-RP3c]

Objectives:
M. 6.3c.1: Define percent.
M. 6.3c.2: Calculate a proportion for missing information.
M. 6.3c.3: Identify a proportion from given information.
M. 6.3c.4: Solve a proportion using part over whole equals percent over 100.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. [6-RP3d]

Objectives:
M. 6.3d.1: Form a ratio.
M. 6.3d.2: Convert like measurement units within a given system.
Example: 120 min = 2 hrs
M. 6.3d.3: Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.

The Number System

Apply and extend previous understandings of multiplication and division to divide by fractions.

4. Interpret and compute quotients of fractions, and solve word problems involving division of fractions, e.g., by using visual fraction models and equations to represent the problem. [6-NS1]

   Examples: Create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$, and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.) How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?

Objectives:
M. 6.4.1: Define fraction (including numerator and denominator), reciprocal, equation, and quotient.
M. 6.4.2: Construct an equation from a given word problem.
M. 6.4.3: Interpret division of fractions by changing division to multiplication.
M. 6.4.4: Demonstrate division of fractions using a visual fraction model.
M. 6.4.5: Demonstrate multiplication of fractions using a visual fraction model.
M. 6.4.6: Discuss the process for multiplying fractions.

Compute fluently with multi-digit numbers and find common factors and multiples.

5. Fluently divide multi-digit numbers using the standard algorithm. [6-NS2]

Objectives:
M. 6.5.1: Define factors and multiples.
M. 6.5.2: Discuss the steps for solving a division problem.
M. 6.5.3: Recognize division and multiplication as inverse operations.
M. 6.5.4: Recall basic division and multiplication facts.

6. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. [6-NS3]

Objectives:
M. 6.6.1: Solve division problems involving multi-digit whole numbers and decimal numbers.
M. 6.6.2: Solve multiplication problems involving multi-digit whole numbers and decimal numbers.
M. 6.6.3: Recall basic multiplication and division facts.
M. 6.6.4: Solve addition and subtraction of multi-digit decimal numbers (emphasis on alignment)
M. 6.6.5: Solve addition and subtraction of multi-digit whole numbers.
M. 6.6.6: Recognize place value of whole numbers and decimals.
M. 6.6.7: Demonstrate addition, subtraction, multiplication, and division of whole numbers and decimals using manipulatives.

7. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. [6-NS4]
   Example: Express 36 + 8 as 4(9 + 2).

Objectives:
M. 6.7.1: Define greatest common factor, least common multiple, and the distributive property.
M. 6.7.2: Design problems using greatest common factor and the distributive property.
M. 6.7.3: Show an understanding of how to solve a problem using the distributive property.
M. 6.7.4: Identify the least common multiple of a given set of numbers.
M. 6.7.5: List multiples of any given whole number.
M. 6.7.6: Identify the greatest common factors of a given set of numbers.
M. 6.7.7: List common factors of given whole numbers.

Apply and extend previous understandings of numbers to the system of rational numbers.

8. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts explaining the measure of centering of 0 in each situation. [6-NS5]

Objectives:
M. 6.8.1: Define integers, positive and negative numbers.
M. 6.8.2: Demonstrate the location of positive and negative numbers on a vertical and horizontal number line.
M. 6.8.3: Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts.
M. 6.8.4: Discuss the measure of centering of 0 in relationship to positive and negative numbers.

9. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. [6-NS6]

Objectives:
M. 6.9.1: Define quadrant, coordinate plane, coordinate axes (x-axis and y-axis), horizontal, vertical, and reflection.
M. 6.9.2: Demonstrate an understanding of an extended coordinate plane.
M. 6.9.3: Draw and label a 4 quadrant coordinate plane.
M. 6.9.4: Draw and extend vertical and horizontal number lines.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., – (–3) = 3, and that 0 is its own opposite. [6-NS6a]

Objectives:
M. 6.9a.1: Discover that the opposite of the opposite of a number is the number itself.
M. 6.9a.2: Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. [6-NS6b]

Objectives:
M. 6.9b.1: Define ordered pairs.
M. 6.9b.2: Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
M. 6.9b.3: Identify which signs indicate the location of a point in a coordinate plane.
M. 6.9b.4: Recall how to plot ordered pairs on a coordinate plane.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. [6-NS6c]

Objectives:
M. 6.9c.1: Define rational number.
M. 6.9c.2: Plot pairs of integers and/or rational numbers on a coordinate plane.
M. 6.9c.3: Arrange integers and /or rational numbers on a horizontal or vertical number line.
M. 6.9c.4: Locate the position of integers and/or rational numbers on a horizontal or vertical number line.
M. 6.9c.5: Identify a rational number as a point on the number line.
M. 6.9c.6: Name the pairs of integers and /or rational numbers of a point on a coordinate plane.

10. Understand ordering and absolute value of rational numbers. [6-NS7]

Objectives:
M. 6.10.1: Define absolute value.
M. 6.10.2: Recall how to order numbers**.**

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. [6-NS7a]
Example: Interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

Objectives:
M. 6.10a.1: Define inequality.
M. 6.10a.2: Explain the inequality of two numbers using their position on a number line in relation to 0.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. [6-NS7b]
Example: Write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.

Objectives:
M. 6.10b.1: Explain how to apply statements of order for rational numbers in real-world contexts.
M. 6.10b.2: Restate the measure of centering of a statement of order for rational numbers in real-world context.
M. 6.10b.3: Write a statement of order for rational numbers in real-world contexts.
M. 6.10b.4: Recall the measure of centering of greater than and less than and their symbols.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. [6-NS7c]
   Example: For an account balance of –30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

Objectives:
M. 6.10c.1: Give examples of the magnitude for a positive or negative quantity in a real-world situations using absolute value.
M. 6.10c.2: Recognize the absolute value of a rational number is its' distance from 0 on a vertical and horizontal number line.

d. Distinguish comparisons of absolute value from statements about order. [6-NS7d]
   Example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.

Objectives:
M. 6.10d.1: Evaluate a statement about order using comparisons of absolute value.
M. 6.10d.2: Recall how to order positive and negative numbers. (Use number line if needed)

11. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. [6-NS8]

Objectives:
M. 6.11.1: Calculate the distances between points having the same first or second coordinate using absolute value.
M. 6.11.2: Interpret graphing points in all four quadrants of the coordinate plane in real-world situations.
M. 6.11.3: Recall how to graph points in all four quadrants of the coordinate plane.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

12. Write and evaluate numerical expressions involving whole-number exponents. [6-EE1]

Objectives:
M. 6.12.1: Define exponent, numerical expression, algebraic expression, variable, base, power, square of a number, and cube of a number.
M. 6.12.2: Compute a numerical expression with exponents.
M. 6.12.3: Restate exponential numbers as repeated multiplication.
M. 6.12.4: Choose the correct value to replace each variable in the expression (Substitution).
M. 6.12.5: Calculate the multiplication of single or multi digit whole numbers.

13. Write, read, and evaluate expressions in which letters stand for numbers. [6-EE2]

Objectives:
M. 6.13.1: Define algebraic expression and variable.
M. 6.13.2: Convert mathematical term to mathematical symbols and numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. [6-EE2a]
   Example: Express the calculation, "Subtract *y* from 5," as $5 - y$.

Objectives:
M. 6.13a.1: Convert mathematical terms to mathematical symbols and numbers (Ex. sum; +, difference; -; product; ●, quotient; ÷).
M. 6.13a.2: Recall different ways to show multiplication and division.

b. Identify parts of an expression using mathematical terms (*sum, term, product, factor, quotient, coefficient*); view one or more parts of an expression as a single entity. [6-EE2b]
   Example: Describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

Objectives:
M. 6.13b.1: Define coefficient and term.
M. 6.13b.2: Match mathematical terms with correct mathematical symbols.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). [6-EE2c]
   Example: Use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

Objectives:
M. 6.13c.1: Calculate an expression in the correct order (Ex. exponents, mult./div. from left to right, and add/sub. from left to right).
M. 6.13c.2: Choose the correct value to replace each variable in the algebraic expression (Substitution).
M. 6.13c.3: Calculate a numerical expression (Ex. V=4 • 4 • 4).
M. 6.13c.4: Recognize the correct order to solve expressions with more than one operation.

14. Apply the properties of operations to generate equivalent expressions. [6-EE3]
   Example: Apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

Objectives:
M. 6.14.1: Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication.
M. 6.14.2: Simplify expressions with parenthesis (Ex. $5(4 + x) = 20 + 5x$).
M. 6.14.3: Combine terms that are alike of a given expression.
M. 6.14.4: Recognize the property demonstrated in a given expression.
M. 6.14.5: Simplify an expression by dividing by the greatest common factor.
   Example: $18x + 6y = 6(3x + y)$.
M. 6.14.6: Determine the greatest common factor.

15. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). [6-EE4]
    Example: The expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y represents.

Objectives:
M. 6.15.1: Define equivalent expressions.
M. 6.15.2: Recognize equivalent expressions.
M. 6.15.3: Substitute for the variable to find the value of a given expression.
M. 6.15.4: Calculate a numerical expression (Ex. $V=4 \bullet 4 \bullet 4$).
M. 6.15.5: Recognize that a variable without a written coefficient is understood to be one.

Reason about and solve one-variable equations and inequalities.

16. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. [6-EE5]

Objectives:
M. 6.16.1: Define equation, solution of an equation, solution of an inequality, and inequality.
M. 6.16.2: Compare and contrast equations and inequalities.
M. 6.16.3: Determine if an inequality is by replacing the variable with a given number.
M. 6.16.4: Determine if an equation is true by replacing the variable with a given number.
M. 6.16.5: Simplify a numerical sentence to determine equivalence.
M. 6.16.6: Recognize the symbols for $=, >, <, \leq$, and $\geq$.

17. Use variables to represent numbers, and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set. [6-EE6]

Objectives:
M. 6.17.1: Define equation and variable.
M. 6.17.2: Set up an equation to represent the given situation, using correct mathematical operations and variables.
M. 6.17.3: Identify the unknown, in a given situation, as the variable.
M. 6.17.4: List given information from the problem.

18. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. [6-EE7]

Objectives:
M. 6.18.1: Define equation and variable.
M. 6.18.2: Solve the equation represented by the real-world situation.
M. 6.18.3: Set up an equation to represent the given situation, using correct mathematical operations and variables.
M. 6.18.4: Identify the unknown variable in a given situation.
M. 6.18.5: List given information from the problem.

19. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. [6-EE8]

Objectives:

M. 6.19.1: Define inequality and solution set of an inequality.
M. 6.19.2: Set up an inequality to represent the given situation, using correct mathematical operations and variables.
M. 6.19.3: Identify solution set for the inequality used to represent the situation.
M. 6.19.4: Graph the solution set on a number line for the inequality used to represent the situation.
M. 6.19.5: Recognize the inequality symbols; <, >.
M. 6.19.6: Construct and label a number line.

Represent and analyze quantitative relationships between dependent and independent variables.

20. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. [6-EE9]

    Example: In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Objectives:

M. 6.20.1: Define dependent variable, independent variable, ordered pairs, input, output, and coordinate plane.
M. 6.20.2: Examine the graph and table to determine any relationship between the variables.
M. 6.20.3: Analyze the pattern represented by the values in the table and develop an equation to express the relationship.
M. 6.20.4: Relate the table and graph to the equation.
M. 6.20.5: Plot independent (input) and dependent (output) values on a coordinate plane.
M. 6.20.6: Create a table of independent and dependent values from the equation.
M. 6.20.7: Draw and label a coordinate plane.
M. 6.20.8: Recall how to draw a number line.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

21. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. [6-G1]

Objectives:
M. 6.21.1: Define area, special quadrilaterals, right triangles, and polygons.
M. 6.21.2: Analyze the area of other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.
M. 6.21.3: Apply area formulas to solve real-world mathematical problems.
M. 6.21.4: Demonstrate how the area of a rectangle is equal to the sum of the area of two equal right triangles.
M. 6.21.5: Explain how to find the area for rectangles.
M. 6.21.6: Select manipulatives to demonstrate how to compose and decompose triangles and other shapes.
M. 6.21.7: Recognize and demonstrate that two right triangles make a rectangle.

22. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. [6-G2]

Objectives:
M. 6.22.1: Define volume, rectangular prism, edge, and formula.
M. 6.22.2: Evaluate the volumes of rectangular prisms in the context of solving real-world and mathematical problems.
M. 6.22.3: Set up V=lwh and V=Bh to find volumes in the context of solving real-world and mathematical problems.
M. 6.22.4: Calculate the volume of a rectangular prism using fractional lengths.
M. 6.22.5: Test the formula V= lwh and V=Bh with the experimental findings.
M. 6.22.6: Experiment with finding the volume using a variety of sizes of rectangular prisms.
M. 6.22.7: Discover the volume of a rectangular prism using manipulatives.
M. 6.22.8: Recall how to multiply fractional numbers.

23. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. [6-G3]

Objectives:
M. 6.23.1: Define vertices**.**
M. 6.23.2: Apply absolute value to find the length of a side joining points with the same first coordinate or the same second coordinate.
M. 6.23.3: Plot points on a Cartesian plane , then connect points for the vertices to sketch a polygon.
M. 6.23.4: Identify ordered pairs.
M. 6.23.5: Recognize polygons.

24. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. [6-G4]

Objectives:
M. 6.24.1: Define three-dimensional figures, surface area, and nets**.**
M. 6.24.2: Evaluate how to apply using surface area of a three-dimensional figure to solving real-world and mathematical problems.
M. 6.24.3: Draw nets to find the surface area of a given three-dimensional figure.
M. 6.24.4: Recall how to calculate the area of a rectangle and triangle.
M. 6.24.5: Select and create a three-dimensional figure using manipulatives.
M. 6.24.6: Identify three-dimensional figures.

Statistics and Probability

Develop understanding of statistical variability.

25. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. [6-SP1]
    Example: "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.

Objectives:
M. 6.25.1: Define statistical question.
M. 6.25.2: Compare and contrast statistical questions and non- statistical questions.

26. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. [6-SP2]

Objectives:
M. 6.26.1: Define distribution and skew.
M. 6.26.2: Describe the shape of a set of data in a given distribution.
M. 6.26.3: Describe the spread of a set of data in a given distribution.
M. 6.26.4: Describe the center of a set of data in a given distribution.
M. 6.26.5: Recall how to read a graph or table.

27. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. [6-SP3]

Objectives:
M. 6.27.1: Define numerical data set, measure of variation, and measure of center.
M. 6.27.2: Relate the measure of variation, of a data set, with the concept of ranGEO.
M. 6.27.3: Relate the measure of the center for a numerical data set with the concept of measure of center.

Summarize and describe distributions.

28. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. [6-SP4]

Objectives:
M. 6.28.1: Define dot plots, upper quartile, lower quartile, median, histograms, and box plots.
M. 6.28.2: Calculate upper quartile median, lower quartile median, overall median, greatest value, and lowest value.
M. 6.28.3: Create box plot using calculations.
M. 6.28.4: Plot data on dot plots and histograms.
M. 6.28.5: Construct and label the display.
M. 6.28.6: Recognize the different types of displays.

29. Summarize numerical data sets in relation to their context, such as by: [6-SP5]

    a. Reporting the number of observations. [6-SP5a]
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. [6-SP5b]
    c. Giving quantitative measures of center (median and/or measure of center) and variability (interquartile range and/or measure of center absolute deviation) as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. [6-SP5c]
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. [6-SP5d]

Objectives:
M. 6.29abcd.1: Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute.
M. 6. 29abcd.2: Explain or give a narrative description of the graph including any overall pattern and any striking deviations from the pattern.
M. 6. 29abcd.3: Compare and contrast the center and variation.
M. 6. 29abcd.4: Graph and label the set of data.
M. 6. 29abcd.5: Organize the data.
M. 6. 29abcd.6: Collect the data.
M. 6. 29abcd.7: Describe how attribute was measured including units of measurement.
M. 6. 29abcd.8: Identify the attribute used to create the numerical set.

GRADE 7

Students will:

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. [7-RP1]

   Example: If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.

Objectives:
M. 7.1.1: Define unit rate, proportions, area, length, and ratio**.**
M. 7.1.2: Recall how to find unit rates using ratios.
M. 7.1.3: Recall the steps used to solve division of fraction problems.
M. 7.1.4: Discuss the measure of centering of ratios.

2. Recognize and represent proportional relationships between quantities. [7-RP2]

Objectives:
M. 7.2.1: Define proportions and proportional relationships.
M. 7.2.2: Demonstrate how to write ratios as a fraction.

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. [7-RP2a]

Objectives:
M. 7.2a.1: Define equivalent ratios and origin.
M. 7.2a.2: Locate the origin on a coordinate plane.
M. 7.2a.3 Show how to graph on Cartesian plane.
M. 7.2a.4: Determine if the graph is a straight line through the origin.
M. 7.2a.4: Use a table or graph to determine whether two quantities are proportional.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. [7-RP2b]

Objectives:
M. 7.2b.1: Define a constant and equations**.**
M. 7.2b.2: Create a table from a verbal description, diagram, or a graph.
M. 7.2b.3: Identify numeric patterns and finding the rule for that pattern.
M. 7.2b.4: Recall how to find unit rate.

c. Represent proportional relationships by equations. [7-RP2c]
Example: If total cost *t* is proportional to the number *n* of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as $t = pn$.

Objectives:
M. 7.2c.1: Recall how to write equations to represent a proportional relationship.
M. 7.2c.2: Discuss the use of variables.

d. Explain what a point (x, y) on the graph of a proportional relationship measure of centers in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. [7-RP2d]

Objectives:
M. 7.2d.1: Define ordered pairs**.**
M. 7.2d.2: Show how to plot points on a Cartesian plane.
M. 7.2d.3: Locate the origin on the coordinate plane.
M. 7.2d.4: Discuss the measure of centering of ratios and unit rates.

3. Use proportional relationships to solve multi-step ratio and percent problems. [7-RP3]
Examples: Sample problems may involve simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

Objectives:
M. 7.3.1: Define interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.
M. 7.3.2: Apply definitions to context in real world problems.
M. 7.3.3: Discuss definitions using real world examples.
M. 7.3.4: Solve proportional problems.
M. 7.3.5: Recall how to find percent and ratios.
M. 7.3.6: Recall steps for solving multi-step problems.

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

4. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. [7-NS1]

Objectives:
M. 7.4.1: Define rational numbers, horizontal, and vertical.
M. 7.4.2: Recall how to extend a horizontal number line.
M. 7.4.3: Recall how to extend a vertical number line.
M. 7.4.4: Demonstrate addition and subtraction of whole numbers using a horizontal or vertical number line.
M. 7.4.5: Give examples of rational numbers.

a. Describe situations in which opposite quantities combine to make 0. [7-NS1a]
Example: A hydrogen atom has 0 charge because its two constituents are oppositely charged.

Objectives:
M. 7.4a.1: Explain the measure of centering of 0 in representing positive and negative quantities**.**
M. 7.4a.2: Locate positive and negative numbers on a number line.
M. 7.4a.3: Recall properties of addition and subtraction.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its' opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. [7-NS1b]

Objectives:
M. 7.4b.1: Define absolute value and additive inverse**.**
M. 7.4b.2: Model addition and subtraction using manipulatives.
M. 7.4b.3: Show addition and subtraction using a number line**.**

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. [7-NS1c]

Objectives:
M. 7.4c.1: Define absolute value and additive inverse.
M. 7.4c.2: Show subtraction as the additive inverse.
M. 7.4c.3: Give examples of the opposite of a given number.
M. 7.4c.4: Show addition and subtraction using a number line**.**

d. Apply properties of operations as strategies to add and subtract rational numbers. [7-NS1d]

Objectives:
M. 7.4d.1: Discuss various strategies for solving real-world and mathematical problems.
M. 7.4d.2: Identify properties of operations for addition and subtraction.
M. 7.4d.3: Recall the steps for solving addition and subtraction of rational numbers.

5. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. [7-NS2]

Objectives:
M. 7.5.1: Recall the steps for solving multiplication and division of fraction problems.
M. 7.5.2: Recall the steps for solving multiplication and division of whole number problems.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. [7-NS2a]

Objectives:
M. 7.5a.1: Define distributive property, rational numbers, product**.**
M. 7.5a.2: Solve problems using the distributive property.
M. 7.5a.3: Recall basic multiplication facts using manipulatives.
M. 7.5a.4: Identify the properties of operations for multiplication.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If *p* and *q* are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts. [7-NS2b]

Objectives:
M. 7.5b.1: Define quotient, divisor, and integer**.**
M. 7.5b.2: Recall the rules for multiplying integers.
M. 7.5b.3: Solve real-world problems.
M. 7.5b.4: Recall the steps of division.

c. Apply properties of operations as strategies to multiply and divide rational numbers. [7-NS2c]

Objectives:
M. 7.5c.1: Discuss various strategies for solving real-world and mathematical problems**.**
M. 7.5c.2: Identify properties of operations for multiplication.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. [7-NS2d]

Objectives:
M. 7.5d.1: Define terminating decimals.
M. 7.5d.2: Give examples of equivalent fractions and decimals.
M. 7.5d.3: Recall the steps for dividing decimals.
M. 7.5d.4: Recall the steps of division.

6. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) [7-NS3]

Objectives:
M. 7.6.1: Discuss various strategies for solving real-world and mathematical problems**.**
M. 7.6.2: Recall steps for solving fractional problems**.**
M. 7.6.3: Identify properties of operations for addition and multiplication.
M. 7.6.4: Recall the rules for multiplication and division of rational numbers.
M. 7.6.5: Recall the rules for addition and subtraction of rational numbers.

Expressions and Equations

Use properties of operations to generate equivalent expressions.

7. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions

Objectives:
M. 7.7.1: Define linear expression, rational, coefficient, and rational coefficient.
M. 7.7.2: Simplify an expression by dividing by the greatest common factor (Ex. 18x + 6y= 6(3x + y).
M. 7.7.3: Simplify expressions with parenthesis (Ex. 5(4 + x) = 20 + 5x).
M. 7.7.4: Recognize the property demonstrated in a given expression.
M. 7.7.5: Combine like terms of a given expression.
M. 7.7.6: Recall how to find the greatest common factor.
M. 7.7.7: Give examples of the properties of operations including distributive, commutative, and associative.

8. Understand that rewriting an expression in different forms in a problem context can shed light on the problem, and how the quantities in it are related. [7-EE2]
   Example: $a + 0.05a = 1.05a$ measure of centers that "increase by 5%" is the same as "multiply by 1.05."

Objectives:
M. 7.8.1: Define expression, equivalent, and equivalent expressions.
M. 7.8.2: Recognize that a variable without a written coefficient is understood to have a coefficient of one.
M. 7.8.3: Recall how to convert mathematical terms to mathematical symbols and numbers and vice versa.
M. 7.8.4: Restate numerical expressions with words.
M. 7.8.5: Recall mathematical terms such as sum, difference, etc.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

9. Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. [7-EE3]

   Examples: If a woman making $25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Objectives:
M. 7.9.1: Define estimation, rational numbers, and reasonable.
M. 7.9.2: Analyze the given word problem to set up a mathematical problem.
M. 7.9.3: Recognize the mathematical operations of rational numbers in any form, including converting between forms. (Ex. 0.25=1/4 =25%)
M. 7.9.4: Recall problem solving methods.
M. 7.9.5: Recognize the rules of operations of positive and negative numbers.
M. 7.9.6: Recognize properties of numbers (Distributive, Associative, Commutative).
M. 7.9.7: Recall mental calculation strategies.
M. 7.9.8: Recall estimation strategies.

10. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. [7-EE4]

Objectives:
M. 7.10.1: Define equation, inequality, and variable.
M. 7.10.2: Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables.
M. 7.10.3: Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations.
M. 7.10.4: Test the found number or number set for accuracy by substitution.
M. 7.10.5: Recall solving one step equations and inequalities.
M. 7.10.6: Recognize properties of numbers (Distributive, Associative, Commutative).

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. [7-EE4a]
   Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Objectives:

M. 7.10a.1: Define equation and variable.
M. 7.10a.2: Set up an equation to represent the given situation, using correct mathematical operations and variables.
M. 7.10a.3: Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations.
M. 7.10a.4: Test the found number for accuracy by substitution.
   Example: Is 5 an accurate solution of $2(x + 5)=12$?
M. 7.10a.5: Identify the unknown, in a given situation, as the variable.
M. 7.10a.6: List given information from the problem.
M. 7.10a.7: Recall solving one-step equations.
M. 7.10a.8: Explain the distributive property.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem. [7-EE4b]
   Example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Objectives:

M. 7.10b.1: Define inequality and variable.
M. 7.10b.2: Set up an inequality to represent the given situation, using correct mathematical operations and variables.
M. 7.10b.3: Calculate a solution set to an inequality by combining like terms, isolating the variable, and/or using inverse operations.
M. 7.10b.4: Test the solution set for accuracy.
M. 7.10b.5: Identify the unknown, of a given situation, as the variable.
M. 7.10b.6: List information from the problem.
M. 7.10b.7: Recall how to graph inequalities on a number line.
M. 7.10b.8: Recall how to solve one step inequalities.

Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

11. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. [7-G1]

Objectives:
M. 7.11.1: Define scale, scale drawings, length, area, and geometric figures**.**
M. 7.11.2: Locate/use scale on a map.
M. 7.11.3: Identify proportional relationships.
M. 7.11.4: Recognize numeric patterns.
M. 7.11.5: Recall how to solve proportions using cross products.

12. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. [7-G2]

Objectives:
M. 7.12.1: Demonstrate how to use a protractor to draw an angle.
M. 7.12.2: Draw segments of a given length using a ruler.
M. 7.12.3: Recognize attributes of geometric shapes.

13. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. [7-G3]

Objectives:
M. 7.13.1: Define two-dimensional figure, three-dimensional figure, plane section, rectangular prism, and rectangular pyramid**.**
M. 7.13.3: Describe the relationship between two- and three-dimensional figures.
M. 7.13.4: Recognize symmetry.
M. 7.13.5: List attributes of three-dimensional figures.
M. 7.13.6: List attributes of two-dimensional figures.

Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

14. Know the formulas for the area and circumference of a circle, and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. [7-G4]

Objectives:
M. 7.14.1: Define circumference, area of a circle, and formula**.**
M. 7.14.2: Identify and label parts of a circle.
M. 7.14.3: Recognize the attributes of a circle.

15. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure. [7-G5]

Objectives:
M. 7.15.1: Define supplementary angles, complementary angles, vertical angles, adjacent angles.
M. 7.15.2: Discuss strategies for solving multi-step problems.
M. 7.15.3: Identify all types of angles.
M. 7.15.4: Identify right angles and straight angles.
M. 7.15.5: Discuss parallel, perpendicular, and intersecting lines.

16. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. [7-G6]

Objectives:
M. 7.16.1: Define volume, surface area, triangles, quadrilaterals, polygons, cubes, and right prisms.
M. 7.16.2: Discuss strategies for solving real-world mathematical problems.
M. 7.16.3: Recall formulas for calculating volume and surface area.
M. 7.16.4: Identify the attributes of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Use random sampling to draw inferences about a population.

17. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. [7-SP1]

Objectives:
M. 7.17.1: Define sample, validity, population, inference, random sampling, statistic, and generalization.
M. 7.17.2: Explain the validity of random sampling.
M. 7.17.3: Differentiate the appropriate sampling method.
M. 7.17.4: Analyze attributes of sample size.
M. 7.17.5: Compare sample size with population to check for validity.
M. 7.17.6: Discuss real world examples of valid sampling and generalizations.
M. 7.17.7: Recall the nature of the attribute, how it was measured, and its unit of measure.

18. Use data from a random sample to draw inferences about a population with an unknown attribute of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. [7-SP2]
    Example: Estimate the measure of center word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Objectives:
M. 7.18.1: Define data, random sampling, population, variation, prediction, estimation, and inference.
M. 7.18.2: Compare and contrast the random sampling data to the population.
M. 7.18.3: Predict an outcome of the entire population based on random samplings.
M. 7.18.4: Collect data from population randomly, choosing same size samples. (Ex. If population is your school, different random samplings should be same number of students)
M. 7.18.5: Recall how to calculate range, outlier, ratio, and proportion.

Draw informal comparative inferences about two populations.

19. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. [7-SP3]
    Example: The measure of center height of players on the basketball team is 10 cm greater than the measure of center height of players on the soccer team, about twice the variability (measure of center absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

Objectives:
M. 7.19.1: Define measure of variability, distribution, and measure of center.
M. 7.19.2: Analyze the skew of the distributions and recognize the difference in measure of center and variability.
M. 7.19.3: Compare the measure of center and measure of variability of two distributions.
M. 7.19.4: Relate the measure of variation with the concept of ranGEO.
M. 7.19.5: Relate the measure of the center with the concept of mean.
M. 7.19.6: Recall how to calculate measure of center and measure of variability.
M. 7.19.7: Discuss how to read and interpret a graph.

20. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. [7-SP4]
    Example: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Objectives:
M. 7.20.1: Define measure of variability, measure of center, and inference.
M. 7.20.2: Compare the measure of center and measure of variability of two numerical data sets.
M. 7.20.3: Recall that center is related to measure of center and measure of variability is related to variation.
M. 7.20.4: Recall how to calculate measure of center and measure of variability.

Investigate chance processes and develop, use, and evaluate probability models.

21. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. [7-SP5]

Objectives:
M. 7.21.1: Define probability and event.
M. 7.21.2: Recall the order of fractions on a number line.
M. 7.21.3: Demonstrate how to compare fractions with different denominators.
M. 7.21.4: Recall how to compare fractions with like denominators.

22. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. [7-SP6]
    Example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

Objectives:
M. 7.22.1: Define probability of chance, outcome, and event.
M. 7.22.2: Recognize the difference between possible outcomes and likely outcomes.
M. 7.22.3: Write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
M. 7.22.4: Recall how to simplify fraction to lowest terms.
M. 7.22.5: Recognize equivalent fractions.

23. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. [7-SP7]

Objectives:
M. 7.23.1: Define probability of chance, probability of events, outcome, and probability of observed frequency.
M. 7.23.2: Compare and contrast probability of chance and probability of observed frequency.
M. 7.23.3: Display all outcomes in a graphic representation (probability model-tree diagram, organized list, table, etc.).
M. 7.23.4: Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
M. 7.23.5: Recall how to simplify fractions to lowest terms.
M. 7.23.6: Recognize equivalent fractions.
M. 7.23.7: Recall how to create a table or graphic display of data.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. [7-SP7a]
   Example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

Objectives:
M. 7.23a.1: Define probability of chance, outcome, and event.
M. 7.23a.2: List all possible outcomes using a graphic representation (probability model-tree diagram, organized list, table, etc.).
M. 7.23a.3: Using the model, count the frequency of the desired outcome.
M. 7.23a.4: Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
M. 7.23a.5: Recall how to simplify fractions to lowest terms.
M. 7.23a.6: Recognize equivalent fractions.
M. 7.23a.7: Recall how to create a table or graphic display of data.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. [7-SP7b]
   Example: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Objectives:
M. 7.23b.1: Define probability of observed frequency, outcome, and event.
M. 7.23b.2: List all actual outcome using a graphic representation (probability model-tree diagram, organized list, table, etc.).
M. 7.23b.3: Using the model, count the frequency of the actual outcome.
M. 7.23b.4: Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
M. 7.23b.5: Recall how to simplify fractions in lowest terms.
M. 7.23b.6: Recognize equivalent fractions.
M. 7.23b.7: Recall how to create a table or graphic display of data.

24. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. [7-SP8]

Objectives:
M. 7.24.1: Define compound events, simulation, frequency, simple events and probability of events.
M. 7.24.2: Discover when to add or multiply events to find probability of compound events.
M. 7.24.3: Choose appropriate model to display outcomes (tree diagram, organized list, or table).
M. 7.24.4: Recall how to find the probability of simple events.
M. 7.24.5: Demonstrate how to add and multiply fractions.
M. 7.24.6: Recall how to obtain a common denominator when adding fractions.
M. 7.24.7: Compute adding fractions with like denominators.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. [7-SP8a]

Objectives:
M. 7.24a.1: Define simple events and compound events.
M. 7.24a.2: Discover when to add or multiply events to find probability of compound events.
M. 7.24a.3: Recall how to find the probability of simple events.
M. 7.24a.4: Demonstrate adding and multiplying fractions.
M. 7.24a.5: Recognize how to obtain a common denominator when adding fractions.
M. 7.24a.6: Recall how to add fractions with like denominators.

b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. [7-SP8b]

Objectives:
M. 7.24b.1: Define compound events.
M. 7.24b.2: Create a tree diagram including all possible outcomes.
M. 7.24b.3: Choose appropriate model to display outcomes (tree diagram, organized list, or table).
M. 7.24b.4: Identify the desired outcomes in model.

c. Design and use a simulation to generate frequencies for compound events. [7-SP8c]
Example: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Objectives:
M. 7.24c.1: Define simulation, frequency, and compound events.
M. 7.24c.2: Discover when to add or multiply events to find probability of compound events.
M. 7.24c.3: Recall how to find the probability of simple events.
M. 7.24c.4: Demonstrate adding and multiplying fractions.
M. 7.24c.5: Recognize how to obtain a common denominator when adding fractions.
M. 7.24c.6: Recall how to add fractions with like denominators.
M. 7.24c.7: Recall how to construct a table.

GRADE 8

Students will:

The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. [8-NS1]

Objectives:
M. 8.1.1: Define rational number and irrational number.
M. 8.1.2: Identify and give examples of rational numbers.
M. 8.1.3: Demonstrate how to convert fractions to decimals.
M. 8.1.4: Recall steps for division of fractions.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). [8-NS2]
   Example: By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Objectives:
M. 8.2.1: Define expressions and approximations.
M. 8.2.2: Identify properties of exponents.
M. 8.2.3: Recall how to compare numbers.
M. 8.2.4: Identify perfect squares and square roots.
M. 8.2.5: Demonstrate how to locate points on a vertical or horizontal number line.
M. 8.2.6: Recall how to estimate.

Expressions and Equations

Work with radicals and integer exponents.

3. Know and apply the properties of integer exponents to generate equivalent numerical expressions. [8-EE1]

   Example: $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

M. 8.3.1: Define exponent, power, coefficient, integers, equivalent, and numerical expression.
M. 8.3.2: Restate negative exponents as positive exponents in the form $1/x^y$.
M. 8.3.3: Recognize to add exponents when multiplying terms with like bases (Property of product of powers).
M. 8.3.4: Recognize to subtract exponents when dividing terms with like bases (Property of quotient of powers).
M. 8.3.5: Compute a numerical expression with positive exponents.
M. 8.3.6: Restate exponential numbers as repeated multiplication.
M. 8.3.7: Compute problems with adding and subtracting integers.

4. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. [8-EE2]

Objectives:
M. 8.4.1: Define square root, cube root, inverse, perfect square, perfect cube, and irrational number.
M. 8.4.2: Recognize the inverse operation of squaring a number is square root and the inverse of cubing a number is cube root.
M. 8.4.3: Restate exponential numbers as repeated multiplication.
M. 8.4.4: Calculate the multiplication of single or multi-digit whole numbers.
M. 8.4.5: Recognize rational and irrational numbers.

5. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. [8-EE3]
   Example: Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

Objectives:
M. 8.5.1: Recognize a fraction as division of the denominator into the numerator.
M. 8.5.2: Recall that when dividing powers of like bases; subtract the exponents (Property of quotient of powers).
M. 8.5.3: Demonstrate how to convert fractions to a decimal.
M. 8.5.4: Recall how to write numbers in scientific notation.
M. 8.5.5: Recall estimation strategies.

6. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. [8-EE4]

M. 8.6.1: Define scientific notation.
M. 8.6.2: Calculate multiplication and division of scientific notation.
M. 8.6.3: Recall properties of exponents.
M. 8.6.4: Discuss the real world application of scientific notation (very large or very small quantities).
M. 8.6.5: Demonstrate difference of scientific notation symbol between paper and calculator.
M. 8.6.6: Recall how to write a number using scientific notation.
M. 8.6.7: Restate exponents as repeated multiplication.

Understand the connections among proportional relationships, lines, and linear equations.

7. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. [8-EE5]
   Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Objectives:
M. 8.7.1: Define proportional relationships, unit rate, and slope.
M. 8.7.2: Demonstrate how to write ratios.
M. 8.7.3: Recall how to solve proportions using cross products.
M. 8.7.4: Demonstrate how to graph on a Cartesian plane.
M. 8.7.5: Recall how to find unit rate.

8. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. [8-EE6]

Objectives:
M. 8.8.1: Define similar triangles, intercept, slope, vertical, horizontal, and origin.
M. 8.8.2: Generate the slope of a line using given ordered pairs.
M. 8.8.3: Analyze the graph to determine the rate of change.
M. 8.8.4: Demonstrate how to plot points on a coordinate plane using ordered pairs from table.
M. 8.8.5: Recall how to complete a function table.
M. 8.8.6: Recognize ordered pairs.
M. 8.8.7: Recognize similar triangles.
M. 8.8.8: Identify intersecting lines.

Analyze and solve linear equations and pairs of simultaneous linear equations.

9. Solve linear equations in one variable. [8-EE7]

Objectives:
M. 8.9.1: Define linear equation and variable.
M. 8.9.2: Recall how to solve equations for a missing variable.
M. 8.9.3: Recall properties of operation for addition and multiplication.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). [8-EE7a]

Objectives:
M. 8.9a.1: Define equivalent equation.
M. 8.9a.2: Recall how to solve equations by using substitution.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions, using the distributive property and collecting like terms. [8-EE7b]

Objectives:
M. 8.9b.1: Define coefficient and distributive property.
M. 8.9b.2: Solve multi-step equations.
M. 8.9b.3: Identify properties of operations.
M. 8.9b.4: Recall how to expand expressions.
M. 8.9b.5: Recall how to solve problems using the distributive property.
M. 8.9b.6: Demonstrate how to simplify equations.

10. Analyze and solve pairs of simultaneous linear equations. [8-EE8]

Objectives:
M. 8.10.1: Define simultaneous.
M. 8.10.2: Recall how to solve linear equations.
M. 8.10.3: Recall properties of operation for addition and multiplication.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersections of their graphs because points of intersection satisfy both equations simultaneously. [8-EE8a]

Objectives:
M. 8.10a.1: Define point of intersection**.**
M. 8.10a.2: Recall how to solve linear equations.
M. 8.10a.3: Demonstrate how to graph on the Cartesian plane.
M. 8.10a.4: Identify ordered pairs.
M. 8.10a.5: Recall how to solve equations by using substitution**.**

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. [8-EE8b]
Example: $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

Objectives:
M. 8.10b.1: Define variables.
M. 8.10b.2: Recall how to estimate.
M. 8.10b.3: Recall how to solve linear equations.
M. 8.10b.4: Demonstrate how to graph solutions to linear equations.
M. 8.10b.5: Recall how to graph ordered pairs on a Cartesian plane.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. [8-EE8c]
   Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Objectives:
M. 8.10c.1: Create a word problem from given information**.**
M. 8.10c.2: Recall how to solve linear equations.
M. 8.10c.3: Explain how to write an equation to solve real-world mathematical problems.

Functions

Define, evaluate, and compare functions.

11. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.) [8-F1]

Objectives:
M. 8.11.1: Define function, ordered pairs, input, output.
M. 8.11.2: Demonstrate how to plot points on a Cartesian plane using ordered pairs.
M. 8.11.3: Recall how to complete input/output tables.
M. 8.11.4: Recognize numeric patterns.

12. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [8-F2]
   Example: Given a linear function represented by a table of values and linear function represented by an algebraic expression, determine which function has the greater rate of change.

Objectives:
M. 8.12.1: Define rate of change.
M. 8.12.2: Recognize linear equations.
M. 8.12.3: Recall how to read/interpret information from a table.
M. 8.12.4: Identify algebraic expressions.
M. 8.12.5: Recall how to name points on a Cartesian plane using ordered pairs.

13. Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. [8-F3]
   Example: The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.

Objectives:
M. 8.13.1: Define linear and nonlinear functions.
M. 8.13.2: Recognize linear equations.
M. 8.13.3: Identify ordered pairs.

Use functions to model relationships between quantities.

14. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*,*y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of linear function in terms of the situation it models and in terms of its graph or a table of values. [8-F4]

Objectives:
M. 8.14.1: Define function, rate of change, and initial value.
M. 8.14.2: Recall how to complete an input/output function table.
M. 8.14.3: Recall how to find the rate of change (slope) in a linear equation.
M. 8.14.4: Recall how to name points from a graph (ordered pairs).

15. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. [8-F5]

Objectives:
M. 8.15.1: Define qualitative, increase, and decrease.
M. 8.15.2: Distinguish the difference between linear and nonlinear functions.
M. 8.15.3: Recall how to plot points on a Cartesian plane.
M. 8.15.4: Identify parts of the Cartesian plane.
M. 8.15.5: Recognize ordered pairs.

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

16. Verify experimentally the properties of rotations, reflections, and translations: [8-G1]

Objectives:
M. 8.16.1: Define rotation, reflection, and translation.
M. 8.16.2: Relate slides to translations.
M. 8.16.3: Relate turns to rotations.
M. 8.16.4: Relate flips to reflections.

a. Lines are taken to lines, and line segments are taken to line segments of the same length. [8-G1a]

Objectives:
M. 8.16a.1: Distinguish between lines and line segments.
M. 8.16a.2: Demonstrate how to measure length.

b. Angles are taken to angles of the same measure. [8-G1b]

Objectives:
M. 8.16b.1: Demonstrate how to use a protractor to measure angles.

c. Parallel lines are taken to parallel lines. [8-G1c]

Objectives:
M. 8.16c.1: Identify parallel lines.

17. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. [8-G2]

Objectives:
M. 8.17.1: Define congruent and sequence.
M. 8.17.2: Recognize translations.
M. 8.17.3: Recognize reflections.
M. 8.17.4: Recognize rotations.
M. 8.17.5: Identify attributes of two-dimensional figures.
M. 8.17.6: Identify congruent figures.

18. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. [8-G3]

Objectives:
M. 8.18.1: Define dilation.
M. 8.18.2: Recall how to find scale factor.
M. 8.18.3: Give examples of scale drawings.
M. 8.18.4: Recognize translations.
M. 8.18.5: Recognize reflections.
M. 8.18.6: Recognize rotations.

19. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. [8-G4]

Objectives:
M. 8.19.1: Define similar.
M. 8.19.2: Recognize dilations.
M. 8.19.3: Recognize translations.
M. 8.19.4: Recognize rotations.
M. 8.19.5: Recognize reflections.
M. 8.19.6: Identify similar figures.

20. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. [8-G5]
    Example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give argument in terms of transversals why this is so.

Objectives:
M. 8.20.1: Define exterior angle and transversal.
M. 8.20.2: Identify attributes of triangles**.**
M. 8.20.3: Identify supplemental angles.
M. 8.20.4: Identify vertical angles**.**

Understand and apply the Pythagorean Theorem.

21. Explain a proof of the Pythagorean Theorem and its converse. [8-G6]

Objectives:
M. 8.21.1: Define Pythagorean Theorem, converse, and proof.
M. 8.21.2: Identify right triangles.
M. 8.21.3: Demonstrate how to find square roots.
M. 8.21.4: Solve problems with exponents.

22. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. [8-G7]

Objectives:
M. 8.22.1: Discuss strategies for solving real-world and mathematical problems.
M. 8.22.2: Solve problems using the Pythagorean Theorem.
M. 8.22.3: Identify right triangles.
M. 8.22.4: Demonstrate how to find square roots.
M. 8.22.5: Solve problems with exponents**.**

23. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. [8-G8]

Objectives:
M. 8.23.1: Recall how to name points on a Cartesian plane using ordered pairs.
M. 8.23.2: Recognize ordered pairs (x, y).
M. 8.23.3: Solve problems using the Pythagorean Theorem.
M. 8.23.4: Identify right triangles.
M. 8.23.5: Demonstrate how to find square roots.
M. 8.23.6: Solve problems with exponents.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

24. Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems. [8-G9]

Objectives:
M. 8.24.1: Define formula, volume, cone, cylinders, spheres, and height.
M. 8.24.2: Discuss the measure of volume and give examples.
M. 8.24.3: Solve problems with exponents.
M. 8.24.4: Recall how to find circumference of a circle.
M. 8.24.5: Identify parts of a circle.

Statistics and Probability

Investigate patterns of association in bivariate data.

25. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. [8-SP1]

Objectives:
M. 8.25.1: Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association.
M. 8.25.2: Describe patterns found in a scatter plot.
M. 8.25.3: Demonstrate how to label and plot information on a scatter plot (dot plot).
M. 8.25.4: Distinguish the difference between positive and negative correlation.
M. 8.25.5: Recall how to describe the spread of the scatter plot (dot plot).

26. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. [8-SP2]

Objectives:
M. 8.26.1: Define scatter plot, outlier, linear, quantitative, line of best fit, and variable.
M. 8.26.2: Analyze scatter plots to determine line of best fit.
M. 8.26.3: Explain how to draw informal inferences from data distributions.
M. 8.26.4: Recall how to summarize numerical data sets in relation to their context.
M. 8.26.5: Recognize the concept of outlier and its relationship to the data distribution.

27. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. [8-SP3]
    Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as measure of centering that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Objectives:
M. 8.27.1: Define slope, intercept, linear, equation, and bivariate.
M. 8.27.2: Recall how to determine the rate of change (slope) from a graph.
M. 8.27.3: Identify the parts of the slope-intercept form of an equation.
M. 8.27.4: Recognize how to read a graph.

28. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. [8-SP4]
    Example: Collect data from students in your class on whether or not they have a curfew on school nights, and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Objectives:

M. 8.28.1: Define relative frequency, bivariate, and frequency.
M. 8.28.2: Design a two-way table.
M. 8.28.3: Analyze a two-way table containing categorical variables.
M. 8.28.4: Calculate relative frequency.
M. 8.28.5: Discuss relative frequency.
M. 8.28.6: Design a table.
M. 8.28.7: Recall how to calculate frequency.
M. 8.28.8: Recall how to collect data.

STANDARDS FOR HIGH SCHOOL MATHEMATICS

Categories of Standards for High School Mathematics

The high school mathematics standards are grouped according to six conceptual categories. These categories provide a coherent view of high school mathematics content. A student's work with functions, for example, crosses a number of traditional course boundaries, potentially up to and including Precalculus. The conceptual categories, as listed below, are described in detail on the following pages.

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Additional Standards for High School Mathematics

High school content standards specify the mathematics that all students should learn and be able to do in order to be college and career ready. Additional mathematics content that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by a plus symbol (+), as in this example:

Example: (+) Represent complex numbers on the complex plane in rectangular and polar form, including real and imaginary numbers.

All standards without a plus symbol (+) are included in the mathematics curriculum for all college- and career-ready students. Some standards with a plus symbol (+) also appear in courses intended for all students.

Modeling Standards for High School Mathematics

The Standards for Mathematical Practice include a standard that requires the modeling of mathematics. Detailed information regarding modeling is located on pages 74-75 of this document. Modeling is best interpreted, not as a collection of isolated topics, but rather having relevance to other standards. Specific modeling standards appear throughout the high school mathematics standards, and they are indicated by an asterisk (*). The asterisk (*) may appear after a particular standard as shown in the example below, or it may appear on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Example: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

ALABAMA OCCUPATIONAL DIPLOMA (AOD) COURSES

The *Curriculum Guide to the Alabama Course of Study: Mathematics* contains the course content for AOD credits. The courses provide students with foundational skills identified in the general education math courses. The courses include essential concepts to equip students with the Algebra and Geometry skills necessary for employment and independent living.

Algebraic Concepts (See Appendix B)

This course provides students with prerequisite algebra skills identified in the general education math courses. The course includes essential concepts to prepare students for Algebraic Essentials A & B.

Algebraic Essentials A & B

Instructional objectives in *Algebra I* proceeded by a solid diamond shape (♦) indicate content required for course credit in *Algebraic Essentials A* for the Alabama Occupational Diploma (AOD).

Instructional objectives in *Algebra I* proceeded by a hollow diamond shape (◊) indicate content required for course credit in *Algebraic Essentials B* for the Alabama Occupational Diploma (AOD).

These courses provide students with foundational skills identified in the Algebra I course. The courses include essential concepts to equip students with the algebra skills necessary for employment and independent living.

Geometry Essentials A & B

Instructional objectives in *Geometry* proceeded by a solid diamond shape (♦) indicate content required for course credit in *Geometry Essentials A* for the Alabama Occupational Diploma (AOD).

Instructional objectives in *Geometry* proceeded by a hollow diamond shape (◊) indicate content required for course credit in *Geometry Essentials B* for the Alabama Occupational Diploma (AOD).

These courses provide students with foundational skills identified in the Geometry course. The courses include essential concepts to equip students with the geometry skills necessary for employment and independent living.

Alabama Occupational Diploma (AOD)
Mathematics Course Sequence

Preferred Course Sequence

Algebraic Essentials A	Aligned to Algebra I A
Algebraic Essentials B	Aligned to Algebra I B
Geometry Essentials A	Aligned to Geometry A
Geometry Essentials B	Aligned to Geometry B

Course Sequence for Students Needing Extra Instruction Prior to Algebraic Essentials

Algebraic Concepts	
Algebraic Essentials A	Aligned to Algebra I A
Algebraic Essentials B	Aligned to Algebra I B
Geometry Essentials A	Aligned to Geometry A

Alabama State Department of Education, Special Education Services, March 2012

ALGEBRA I

The Algebra I course builds on foundational mathematics content learned by students in Grades K-8 by expanding mathematics understanding to provide students with a strong mathematics education. Content is designed to engage students in a variety of mathematical experiences that include the use of reasoning and problem-solving skills, which may be applied to life situations beyond the classroom setting. This course serves as the cornerstone for all high school mathematics courses; therefore, all subsequent mathematics courses require student mastery of the Algebra I content standards.

Algebra I is one of the courses required for all students. School systems may offer Algebra I and Algebra IA and Algebra IB.

Content standards 3, 4, 5, 6, 7, 7a, 15, 16, 21, 22, 24, 25, 26, 29, 30, 30a, 37, 37b, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, and 50 must be taught in the Algebra IA course.

Content standards 1, 2, 7b, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 23, 27, 28, 30b, 30c, 31, 32, 33, 34, 35, 36, 37a, 37c, and 38 must be taught in the Algebra IB course. Systems offering Algebra I in the eighth grade have the responsibility of ensuring that all Algebra I course content standards and Grade 8 course content standards be included in instruction.

Students will:

NUMBER AND QUANTITY

The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. [N-RN1]
   Example: We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

Objectives:

ALG1-B.1.1: Define exponent, integer, rational number, and radicals.
ALG1-B.1.2: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values.
ALG1-B.1.3: Use notation for radicals in terms of rational exponents.
◊ **ALG1-B.1.4:** Apply the properties of integer exponents to generate equivalent numerical expressions.
◊ **ALG1-B.1.5:** Know the properties of integer exponents.
◊ **ALG1-B.1.6:** Write numerical expressions involving whole-number exponents.
◊ **ALG1-B.1.7:** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. [N-RN2]

Objectives:
ALG1-B.2.1: Rewrite expressions involving radicals using the properties of exponents.
ALG1-B.2.2: Rewrite expressions involving rational exponents using the properties of exponents.
◊ **ALG1-B.2.3:** Recognize the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. [N-RN3]

Objectives:
ALG1-A.3.1: Define rational and irrational numbers.
ALG1-A.3.2: Identify the product of a nonzero rational number and an irrational number as irrational.
ALG1-A.3.3: Identify the sum of a rational number and an irrational number is irrational.
ALG1-A.3.4: Discuss why the product of two rational numbers is rational.
ALG1-A.3.5: Discuss why the sum of two rational numbers is rational.
ALG1-A.3.6: Describe the properties of addition and multiplication.
ALG1-A.3.7: Apply properties of fractions to add, subtract, multiply, and divide rational numbers.
ALG1-A.3.8: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Quantities*

Reason quantitatively and use units to solve problems. (*Foundation for work with expressions, equations, and functions.)*

4. Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. [N-Q1]

Objectives:
ALG1-A.4.1: Interpret the scale and the origin in data displays.
♦ **ALG1-A.4.2:** Choose the scale and the origin in graphs.
♦ **ALG1-A.4.3:** Interpret units consistently in formulas.
♦ **ALG1-A.4.4:** Choose units consistently in formulas.
♦ **ALG1-A.4.5:** Use units as a way to guide the solution of multistep problems.
♦ **ALG1-A.4.6:** Use units as a way to understand problems.
♦ **ALG1-A.4.7:** Convert between units of measurement within the same system.

5. Define appropriate quantities for the purpose of descriptive modeling. [N-Q2]

Objectives:
♦ **ALG1-A.5.1:** Define units of measurement.
♦ **ALG1-A.5.2:** Identify appropriate units of measure to best describe a real-world application.

6. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. [N-Q3]

Objectives:
ALG1-A.6.1: Recognize the limitations for each type of measurement tool.
♦ **ALG1-A.6.2:** Determine the level of precision needed for real-world measurements.
♦ **ALG1-A.6.3:** Relate how rounding effects the accuracy of the measurement.

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Linear, exponential, quadratic.)*

7. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]

Objectives:
♦ **ALG1-A.7.1:** Define linear, quadratic and exponential functions.
ALG1-A.7.2: Classify an expression as linear, quadratic or exponential from a table.
♦ **ALG1-A.7.3:** Classify an expression as linear, quadratic or exponential from an equation.
♦ **ALG1-A.7.4:** Classify an expression as linear, quadratic or exponential from a graph.

a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]

Objectives:
♦ **ALG1-A.7a.1:** Define terms, factors, and coefficients.
ALG1-A.7a.2: Identify factors in linear, exponential and quadratic expressions.
ALG1-A.7a.3: Identify coefficients in linear, exponential and quadratic expressions.
ALG1-A.7a.4: Identify terms in linear, exponential and quadratic expressions.
♦ **ALG1-A.7a.5:** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient).

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Objectives:
ALG1-B.7b.1: Recognize one or more parts of an exponential expression as a single entity.
ALG1-B.7b.2: Recognize one or more parts of a quadratic expression as a single entity.
◊ **ALG1-B.7b.3:** Recognize one or more parts of a linear expression as a single entity.

8. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
   Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Objectives:
ALG1-B.8.1: Rewrite an exponential expression in an alternative way.
ALG1-B.8.2: Rewrite a quadratic expression in an alternative way.
◊ **ALG1-B.8.3:** Rewrite a linear expression in an alternative form.
ALG1-B.8.4: Understand that rewriting an expression in different forms in a problem context can shed light on the problem.
ALG1-B.8.5: Recall properties of exponents.

Write expressions in equivalent forms to solve problems. ***(Quadratic and exponential.)***

9. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* [A-SSE3]

Objectives:
ALG1-B.9.1: Convert an expression to an alternative format.
ALG1-B.9.2: Recognize the best format for a specific application.
ALG1-B.9.3: Match equivalent expressions written in different formats.

   a. Factor a quadratic expression to reveal the zeros of the function it defines. [A-SSE3a]

Objectives:
◊ **ALG1-B.9a.1:** Define factor, quadratic expression and zero product property.
ALG1-B.9a.2: Factor a quadratic expression.
◊ **ALG1-B.9a.3:** Use the zero product property to reveal the zeros in the function.
ALG1-B.9a.4: Solve a two-step equation.
ALG1-B.9a.5: Solve a one-step equation.
ALG1-B.9a.6: Determine the Greatest Common Factor (GCF).

   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. [A-SSE3b]

Objectives:
◊ **ALG1-B.9b.1:** Define maximum and minimum value.
ALG1-B.9b.2: Explain the steps for completing the square.
ALG1-B.9b.3: Given a quadratic expression in which the square has already been completed, determine the maximum or minimum values.

   c. Determine a quadratic equation when given its graph or roots. 

Objectives:
◊ **ALG1-B.9c.1:** Define roots.
◊ **ALG1-B.9c.2:** Find the equation using the distributive property.
◊ **ALG1-B.9c.3:** Locate and identify the roots on a graph using the *x*-intercepts.
◊ **ALG1-B.9c.4:** Take given roots and convert into a one-step equation set equal to zero.

d. Use the properties of exponents to transform expressions for exponential functions. [A-SSE3c]
Example: The expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Objectives:

◊ **ALG1-B.9d.1:** Define multiplicative inverse.
ALG1-B.9d.2: Apply the property of multiplicative inverse.
ALG1-B.9d.3: Identify the property of exponents.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. ***(Linear and quadratic.)***

10. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Objectives:

◊ **ALG1-B.10.1:** Define monomial, term, binomial, trinomial and polynomial.
ALG1-B.10.2: Multiply polynomial expressions (quadratic).
◊ **ALG1-B.10.3:** Multiply polynomial expressions (linear).
◊ **ALG1-B.10.4:** Subtract polynomial expressions.
◊ **ALG1-B.10.5:** Add polynomial expressions.
ALG1-B.10.6: Use order of operations to evaluate and simplify algebraic and numerical expressions.
ALG1-B.10.7: Identify the terms in a polynomial expressions.
ALG1-B.10.8: Explain the distributive property.

Creating Equations*

Create equations that describe numbers or relationships. ***(Linear, quadratic, and exponential (integer inputs only); for Standard 13, linear only.)***

11. Create equations and inequalities in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

Objectives:

◊ **ALG1-B.11.1:** Define equation, expression, variable, equality and inequality.
ALG1-B.11.2: Create inequalities with one variable (Exponential, Quadratic, Linear).
ALG1-B.11.3: Create equalities with one variable (Exponential, Quadratic, Linear).
ALG1-B.11.4: Solve two-step equations and inequalities.
ALG1-B.11.5: Solve one-step equations and inequalities using the four basic operations.
◊ **ALG1-B.11.6:** Compare and contrast equations and inequalities.
ALG1-B.11.7: Recognize inequality symbols including $>$, $<$, $\geq$, and $\leq$.

12. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

Objectives:

◊ **ALG1-B.12.1:** Define ordered pair and coordinate plane.
ALG1-B.12.2: Create equations with two variables (exponential, quadratic and linear).
ALG1-B.12.3: Graph equations on coordinate axes with labels and scales (exponential, quadratic, and linear).
ALG1-B.12.4: Identify an ordered pair and plot it on the coordinate plane.

13. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. [A-CED3]
Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

Objectives:

◊ **ALG1-B.13.1:** Define systems of equations, constraints, viable solution, and nonviable solution.
ALG1-B.13.2: Create a system of equations or inequalities to represent the given constraints (linear).
ALG1-B.13.3: Create an equation or inequality to represent the given constraints (linear).
◊ **ALG1-B.13.4:** Determine if a solution to a system of equations or inequalities is viable or nonviable.
◊ **ALG1-B.13.5:** Determine if there is one solution, infinite solutions, or no solutions to a system of equations or inequalities.

14. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Objectives:

ALG1-B.14.1: Solve a familiar literal equation for a specific variable.
◊ **ALG1-B.14.2:** Identify and isolate a specific variable.
◊ **ALG1-B.14.3:** Demonstrate the process of solving multi-step equations.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning. ***(Master linear; learn as general principle.)***

15. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. [A-REI1].

Objectives:

ALG1-A.15.1: List the properties involved in solving a multi-step equation using deductive reasoning.
♦ **ALG1-A.15.2:** Solve a multi-step equation using the properties, assuming that the original equation has a solution.
♦ **ALG1-A.15.3:** Use square root and cube root symbols to represent solutions to equations of the form $x^2=p$ p and $x^3= p$, where p is a positive rational number.
♦ **ALG1-A.15.4:** Evaluate square roots and cube roots.
ALG1-A.15.5: Solve a one-step equation using the appropriate property.
ALG1-A.15.6: Identify the properties of real numbers.
ALG1-A.15.7: Identify the procedures for performing order of operations.

Solve equations and inequalities in one variable. ***(Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions.)***

16. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. [A-REI3]

Objectives:

ALG1-A.16.1: Define coefficient and variable.
ALG1-A.16.2: Solve a multi-step equation or inequality.
ALG1-A.16.3: Solve a one-step equation or inequality.
ALG1-A.16.4: Identify the procedures for performing order of operations.

17. Solve quadratic equations in one variable. [A-REI4]

Objectives:

◊ **ALG1-B.17.1:** Define quadratic equation and zero product property.
ALG1-B.17.2: Solve two-step equations using addition and subtraction that are set equal to zero.
ALG1-B.17.3: Solve one-step equations using addition and subtraction that are set equal to zero.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. [A-REI4a]

Objectives:

◊ **ALG1-B.17a.1:** Define completing the square.
ALG1-B.17a.2: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2= q$ that has the same solutions.
ALG1-B.17a.3: Derive the quadratic formula from the form $(x – p)^2= q$.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b. [A-REI4b]

Objectives:

◊ **ALG1-B.17b.1:** Define quadratic formula, factoring, square root, complex number, and real number.
ALG1-B.17b.2: Solve quadratic equations by completing the square.
◊ **ALG1-B.17b.3:** Solve quadratic equations by the quadratic formula.
◊ **ALG1-B.17b.4:** Solve quadratic equations by factoring.
◊ **ALG1-B.17b.5:** Solve quadratic equations by taking square roots (e.g., for $x^2=49$).
ALG1-B.17b.6: Recognize when the quadratic formula gives complex solutions.
ALG1-B.17b.7: Write complex solutions as $a \pm bi$ for real numbers a and b.

Solve systems of equations. ***(Linear-linear and linear-quadratic.)***

18. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. [A-REI5]

Objectives:

◊ **ALG1-B.18.1:** Define the elimination process and the substitution process for solving systems of equations.
◊ **ALG1-B.18.2:** Use the elimination process to solve systems of equations.
◊ **ALG1-B.18.3:** Use the substitution process to solve systems of equations.
◊ **ALG1-B.18.4:** Evaluate an expression in two variables for given values.
◊ **ALG1-B.18.5:** Evaluate an expression in one variable for given values.

19. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. [A-REI6]

Objectives:

◊ **ALG1-B.19.1:** Define system of linear equations.
◊ **ALG1-B.19.2:** Verify an equation in two variables for a given ordered pair.
◊ **ALG1-B.19.3:** Identify the point of intersection given graphs of two non-parallel lines in the coordinate plane.
◊ **ALG1-B.19.4:** State the formulas for slope-intercept form, point-slope form, and standard form of a line.
◊ **ALG1-B.19.5:** Construct graphs of common relations, including x = constant, y = constant, $y = x$, and $y = mx + b$.

20. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. [A-REI7]
Example: Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Objectives:

ALG1-B.20.1: Use the substitution method to replace a variable in the quadratic equation.
◊ **ALG1-B.20.2:** Solve for the variables in a system of equations. (Algebraically)**.**
◊ **ALG1-B.20.3:** Graph a quadratic equation.
◊ **ALG1-B.20.4:** Graph a linear equation.
◊ **ALG1-B.20.5:** Identify the point(s) of intersection when given graphs.

Represent and solve equations and inequalities graphically. ***(Linear and exponential; learn as general principle.)***

21. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [A-REI10]

Objectives:

♦ **ALG1-A.21.1:** Understand that the graph of an equation is the solution of an equation.
♦ **ALG1-A.21.2:** Graph a linear equation and use the graph to determine the solution set.
♦ **ALG1-A.21.3:** Use a given graph to determine the solution set.
ALG1-A.21.4: Plot given points from a table.

22. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Objectives:

♦ **ALG1-A.22.1:** Define function, function notation, linear, polynomial, rational, absolute value, exponential, and logarithmic functions, and transitive property.
ALG1-A.22.2: Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations.
ALG1-A.22.3: Find solutions to the equations $y = f(x)$ and $y = g(x)$ using the graphing calculator.
♦ **ALG1-A.22.4:** Solve equations for y.
ALG1-A.22.5: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations.

23. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. [A-REI12]

Objectives:

◊ **ALG1-B.23.1:** Define the half-plane as the shaded region.
◊ **ALG1-B.23.2:** Determine the intersecting shaded region is the solution to the system.
◊ **ALG1-B.23.3:** Graph the lines of the systems and shade the appropriate region.
◊ **ALG1-B.23.4:** Determine the shaded region is the solution to the inequality.
◊ **ALG1-B.23.5:** Graph an inequality and shade the appropriate region.
◊ **ALG1-B.23.6:** Determine whether a line should be solid or dotted, depending on the inequality symbol.
ALG1-B.23.7: Recognize inequality symbols $>$, $<$, $\geq$, $\leq$.

FUNCTIONS

Interpreting Functions

Understand the concept of a function and use function notation. ***(Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences.)***

24. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the ranGEO. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). [F-IF1]

Objectives:

♦ **ALG1-A.24.1:** Define domain, range, relation, function, table of values, input, and output.
ALG1-A.24.2: Understand the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
ALG1-A.24.3: Understand that a function is a rule that assigns to each input exactly one output.
♦ **ALG1-A.24.4:** Identify the equation of a function, given its graph.
ALG1-A.24.5: Find the range of a function given its domain.
♦ **ALG1-A.24.6:** Recognize that *f(x)* and *y* are the same.

25. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. [F-IF2]

Objectives:

♦ **ALG1-A.25.1:** Define function notation.
ALG1-A.25.2: Translate a simple word problem into function notation.
♦ **ALG1-A.25.3:** Evaluate function when given *x*-values.

26. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. [F-IF3]
    Example: The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Objectives:

ALG1-A.26.1: Define sequences and recursively-defined sequences.
ALG1-A.26.2: Recognize that sequences are functions whose domain is the set of all positive integers and zero.

Interpret functions that arise in applications in terms of the context. (*Linear, exponential, and quadratic.*)

27. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** [F-IF4]

Objectives:

ALG1-B.27.1: Define intercepts, intervals, relative maxima, relative minima, symmetry, end behavior, and periodicity.
ALG1-B.27.2: For a function that models a relationship between two quantities, find the periodicity.
ALG1-B.27.3: For a function that models a relationship between two quantities, find the end behavior.
ALG1-B.27.4: For a function that models a relationship between two quantities, find the symmetry.
ALG1-B.27.5: For a function that models a relationship between two quantities, find the intervals where the function is increasing, decreasing, positive, or negative**.**
ALG1-B.27.6: For a function that models a relationship between two quantities, find the relative maxima and minima.
◊ **ALG1-B.27.7:** For a function that models a relationship between two quantities, find the *x* and *y* intercepts.

28. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Objectives:

◊ **ALG1-B.28.1:** Define domain, range, relation, function, table of values, and mappings.
◊ **ALG1-B.28.2:** Determine the appropriate domain for a given function.
ALG1-B.28.3: Identify functions from information in tables, sets of ordered pairs, and mappings.
◊ **ALG1-B.28.4:** Translate verbal phrases into a function.
◊ **ALG1-B.28.5:** Arrange data given as ordered pairs into a table and a table of values into ordered pairs.
ALG1-B.28.6: Identify the *x* and *y* values in an ordered pair.

29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Objectives:

♦ **ALG1-A.29.1:** Define average rate of change as slope.
ALG1-A.29.2: Estimate the rate of change from a graph (rise/run).
ALG1-A.29.3: Interpret the average rate of change.
♦ **ALG1-A.29.4:** Calculate the average rate of change.
♦ **ALG1-A.29.5:** Compute the slope of a line given two ordered pairs.
♦ **ALG1-A.29.6:** Identify the slope, given slope-intercept form.

Analyze functions using different representations. ***(Linear, exponential, quadratic, absolute value, step, piecewise-defined.)***

30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]

Objectives:
ALG1-A.30.1: Define piecewise-defined functions and step functions.
ALG1-A.30.2: Graph functions expressed symbolically by hand in simple cases.
ALG1-A.30.3: Graph functions expressed symbolically using technology for more complicated cases.

a. Graph linear and quadratic functions, and show intercepts, maxima, and minima. [F-IF7a]

Objectives:
ALG1-A.30a.1: Graph quadratic functions showing maxima and minima.
ALG1-A.30a.2: Graph quadratic functions showing intercepts.
♦ **ALG1-A.30a.3:** Graph linear functions showing intercepts.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]

Objectives:
◊ **ALG1-B.30b.1:** Define square root, cube root, and absolute value function.
ALG1-B.30b.2: Graph piecewise-defined functions.
ALG1-B.30b.3: Graph step functions.
ALG1-B.30b.4: Graph cube root functions.
ALG1-B.30b.5: Graph square root functions.
◊ **ALG1-B.30b.6:** Graph absolute value functions.

c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

Objectives:
◊ **ALG1-B.30c.1:** Define exponential function, logarithmic function, trigonometric function**,** intercepts, end behavior, period, midline and amplitude.
ALG1-B.30c.2: Graph logarithmic functions showing intercepts and end behavior.
ALG1-B.30c.3: Graph exponential functions showing intercepts and end behavior.
ALG1-B.30c.4: Graph trigonometric functions showing period, midline and amplitude.

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

Objectives:
ALG1-B.31.1: Write a function defined by an expression in different but equivalent forms to reveal different properties of the function.
ALG1-B.31.2: Write a function defined by an expression in different but equivalent forms to explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. [F-IF8a]

Objectives:

ALG1-B.31a.1: Interpret the zeros, extreme values, and symmetry of the graph in terms of a context.
ALG1-B.31a.2: Use the process of completing the square in a quadratic function to show symmetry of the graph, extreme values, and zeros.
ALG1-B.31a.3: Use the process of factoring in a quadratic function to show symmetry of the graph, extreme values, and zeros.
ALG1-B.31a.4: Use completing the square to solve a quadratic equation.
◊ **ALG1-B.31a.5:** Use factoring to solve a quadratic equation.

b. Use the properties of exponents to interpret expressions for exponential functions. [F-IF8b]
Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.

Objectives:

◊ **ALG1-B.31b.1:** Define exponential growth and decay.
ALG1-B.31b.2: Interpret exponential functions.
ALG1-B.31b.3: Relate the properties of exponents.
◊ **ALG1-B.31b.4:** Classify an exponential function as growth or decay.
Example: $y = e^{-kt}$ (decay) and $y = e^{kt}$ (growth) Positive = growth and negative = decay.
ALG1-B.31b.5: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.
◊ **ALG1-B.31b.6:** Apply properties of operations to calculate numbers in any form converting between forms as appropriate.

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Objectives:

ALG1-B.32.1: Compare properties of two functions each represented in a different way.
ALG1-B.32.2: Identify properties of functions defined algebraically.
ALG1-B.32.3: Identify properties of functions defined by verbal description.
ALG1-B.32.4: Identify properties of functions defined graphically.
◊ **ALG1-B.32.5:** Identify properties of functions defined numerically in tables.
Example:

x	y (Linear)
1	2
2	4
3	6

x	y (Quadratic)
1	1
2	4
3	9

Building Functions

Build a function that models a relationship between two quantities. ***(For standards 33 and 34, linear, exponential, and quadratic.)***

33. Write a function that describes a relationship between two quantities.* [F-BF1]

Objectives:

ALG1-B.33.1: Write a function that describes a relationship between two quantities. (Exponential)

ALG1-B 33.2: Write a function that describes a relationship between two quantities. (Quadratic)

◊ **ALG1-B.33.3:** Write a function that describes a relationship between two quantities. (Linear)

a. Determine an explicit expression, a recursive process, or steps for calculation from a context. [F-BF1a]

Objectives:

◊ **ALG1-B.33a.1:** Define explicit expressions and recursive process.

ALG1-B.33a.2: Determine an explicit expression from a context.

Example:

Explicit	Recursive
$a_n = 3n + 2, n = 1$	$a_0 = 5$
$a_1 = 3(1) + 2$	$a_{n+1} = a_n + 3$
$a_1 = 5$	$a_{0+1} = a_0 + 3$
	$a_1 = 5 + 3$
	$a_1 = 8$
	$a_{1+1} = a_1 + 3$
	$a_2 = 8 + 3$
	$a_2 = 11\ldots$

ALG1-B.33a.3: Determine a recursive process from a context.

ALG1-B.33a.4: Evaluate a linear expression from a context (word problem).

b. Combine standard function types using arithmetic operations. [F-BF1b]

Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Objectives:

◊ **ALG1-B.33b.1:** Define standard function types as exponential, quadratic, and linear.

ALG1-B.33b.2: Combine standard functions by dividing a constant function.

ALG1-B.33b.3: Combine standard functions by multiplying a constant function.

◊ **ALG1-B.33b.4:** Combine standard functions by subtracting a constant function.

◊ **ALG1-B.33b.5:** Combine standard functions by adding a constant function.

34. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* [F-BF2]

Objectives:

ALG1-B.34.1: Define arithmetic sequence, geometric sequence, recursive sequence, and explicitly-defined sequence.

Example:

x	y
1	2
2	4
3	6

(Linear)

4-2=2, 6-4=2, so 2 is common difference.

x	y
0	1
1	2
2	4
3	8

(Exponential)

2/1 = 2; 4/2 = 2; 8/4 = 2, so 2 is common quotient

ALG1-B.34.2: In writing geometric sequences, translate between recursive and explicit formula forms.
ALG1-B.34.3: In writing arithmetic sequences, translate between recursive and explicit formula forms.
ALG1-B.34.4: Use geometric sequences to model situations.
ALG1-B.34.5: Use arithmetic sequences to model situations.
ALG1-B.34.6: Write geometric sequences with an explicit formula.
ALG1-B.34.7: Write geometric sequences recursively.
ALG1-B.34.8: Write arithmetic sequences with an explicit formula.
ALG1-B.34.9: Write arithmetic sequences recursively.

Build new functions from existing functions. ***(Linear, exponential, quadratic, and absolute value; for standard 36a, linear only)***

35. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

Objectives:

ALG1-B.35.1: Recognize even and odd functions from algebraic expressions for them.
ALG1-B.35.2: Recognize even and odd functions from their graphs.
ALG1-B.35.3: Experiment with various cases of functions and illustrate an explanation of the effects on the graph using technology. (Exponential, Quadratic, Absolute value)
ALG1-B.35.4: Find the value of k given the graphs of $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$.
ALG1-B.35.5: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

36. Find inverse functions. [F-BF4]

Objectives:

◊ **ALG1-B.36.1:** Define inverse function.
ALG1-B.36.2: Find inverse functions for a function. (Exponential)
ALG1-B.36.3: Find inverse functions for a function. (Quadratic)
ALG1-B.36.4: Find inverse functions for a function. (Absolute Value)
◊ **ALG1-B.36.5:** Find inverse functions for a function. (Linear)
◊ **ALG1-B.36.6:** Apply the substitution principle.
ALG1-B.36.7: Solve a multi-step equation.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Objectives:

ALG1-B.36a.1: Solve an equation of the form $f(x) = c$ for a simple linear function f that has an inverse.
◊ **ALG1-B.36a.2:** Write an expression for the inverse of a simple linear function f of the form $f(x) = c$.
◊ **ALG1-B.36a.3:** Apply the substitution principle.
ALG1-B.36a.4: Solve a multi-step equation.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems.

37. Distinguish between situations that can be modeled with linear functions and with exponential functions. [F-LE1]

Objectives:

♦ **ALG1-A.37.1:** Define linear function and exponential function.
♦ **ALG1-A.37.2:** Distinguish between graphs of a line and an exponential function.
♦ **ALG1-A.37.3:** Identify the graph of an exponential function.
ALG1-A.37.4: Identify the graph of a line.
ALG1-A.37.5: Plot points on a coordinate plane from a given table of values.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. [F-LE1a]

Objectives:

ALG1-B.37a.1: Divide each y-value in a table of values by its successive y-value to determine if the quotients are the same, to prove an exponential function.
◊ **ALG1-B.37a.2:** Subtract each y-value in a table of values by its successive y-value to determine if the differences are the same, to prove a linear function.
ALG1-B.37a.3: Apply rules for adding, subtracting, multiplying, and dividing integers.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. [F-LE1b]

Objectives:

♦ **ALG1-A.37b.1:** Define constant rate of change as slope.
♦ **ALG1-A.37b.2:** Subtract each y-value in a table of values by its successive y-value to determine if the differences are the same, to prove a linear function.
♦ **ALG1-A.37b.3:** Recognize the calculated difference is the constant rate of change.
ALG1-A.37b.4: Apply rules for adding, subtracting, multiplying, and dividing integers.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. [F-LE1c]

Objectives:

◊ **ALG1-B.37c.1**: Define exponential growth and decay.
ALG1-B.37c.2: Divide each *y*-value in a table of values by its successive *y*-value to determine if the quotients are the same, to prove an exponential function.
ALG1-B.37c.3: Apply the rules of multiplication and division of integers.

38. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). [F-LE2]

Objectives:

◊ **ALG1-B.38.1:** Define arithmetic sequence, geometric sequence, and input-output pairs.
ALG1-B.38.2: Construct exponential functions, including geometric sequences, given a description of a relationship.
ALG1-B.38.3: Construct exponential functions, including geometric sequences, given a graph.
ALG1-B.38.4: Construct exponential functions, including geometric sequences, given two input-output pair (including reading these from a table).
◊ **ALG1-B.38.5:** Write an equation of a line, given a chart.
◊ **ALG1-B.38.6:** Write an equation of a line, given a graph.
◊ **ALG1-B.38.7:** Write an equation of a line, given two ordered pairs.

39. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. [F-LE3]

Objectives:

♦ **ALG1-A.39.1:** Define a polynomial function and parabola.
ALG1-A.39.2: Observe that an exponential function increases more rapidly than a line, a parabola, or a polynomial function by using a table.
ALG1-A.39.3: Observe that an exponential function increases more rapidly than a line, a parabola, or a polynomial function by using a graph.
ALG1-A.39.4: Compare the y-values by looking at the same x-value in a variety of tables or graphs.
Example:

Linear		Quadratic		Exponential	
x	*y*	*x*	*y*	*x*	*y*
4	4	4	16	4	16
5	5	5	25	5	32
6	6	6	36	6	64

Constant *x*-values, but *y*-values vary depending on the type of function, with exponential having the greatest increase.

Interpret expressions for functions in terms of the situation they model. ***(Linear and exponential of form $f(x) = b^x + k$.)***

40. Interpret the parameters in a linear or exponential function in terms of a context. [F-LE5]

Objectives:

- **ALG1-A.40.1:** Define *b* as growth or decay factor in the context of an exponential problem.
- **ALG1-A.40.2:** Define *k* as the initial amount in the context of an exponential problem.
- **ALG1-A.40.3:** Define *m* as the rate of change in the context of a linear problem.
- **ALG1-A.40.4:** Define *b* as the initial amount in the context of a linear problem.
- **ALG1-A.40.5:** Recall the formula of an exponential function.
- **ALG1-A.40.6:** Recall the slope-intercept form of a linear function.

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

41. Represent data with plots on the real number line (dot plots, histograms, and box plots). [S-ID1]

Objectives:

- **ALG1-A.41.1:** Define dot plots, histograms, and box plots.
- **ALG1-A.41.2:** Represent data with plots on the real number line, using box plots.
- **ALG1-A.41.3:** Represent data with plots on the real number line, using histograms.
- **ALG1-A.41.4:** Represent data with plots on the real number line, using dot plots.
- **ALG1-A.41.5:** Plot points using given data.

42. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. [S-ID2]

Objectives:

- **ALG1-A.42.1:** Define center, mean, median, spread, interquartile range, standard deviation, and data set.
- **ALG1-A.42.2:** Calculate the interquartile range of two or more different data sets.
- **ALG1-A.42.3:** Calculate the standard deviation of two or more different data sets.
- **ALG1-A.42.4:** Compare the spread.
- **ALG1-A.42.5:** Calculate the mean of two or more different data sets.
- **ALG1-A.42.6:** Calculate the median of two or more different data sets.
- **ALG1-A.42.7:** Compare the center.
- **ALG1-A.42.8:** Organize data sets in either increasing or decreasing values.

43. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). [S-ID3]

Objectives:

♦ **ALG1-A.43.1:** Define outliers.
ALG1-A.43.2: Compare differences in shape in the context of the data sets, accounting for possible effects of outliers.
ALG1-A.43.3: Compare differences in center in the context of the data sets, accounting for possible effects of outliers.
ALG1-A.43.4: Compare differences in spread in the context of the data sets, accounting for possible effects of outliers.

Summarize, represent, and interpret data on two categorical and quantitative variables. ***(Linear focus, discuss general principle.)'***

44. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. [S-ID5]

Objectives:

♦ **ALG1-A.44.1:** Define categorical data, two-way frequency table, relative frequency, joint frequency, marginal frequency, and conditional relative frequency.
ALG1-A.44.2: Recognize possible associations and trends in the data.
ALG1-A.44.3: Interpret conditional relative frequencies in the context of the data.
ALG1-A.44.4: Interpret marginal frequencies in the context of the data.
ALG1-A.44.5: Interpret joint frequencies in the context of the data.
ALG1-A.44.6: Interpret relative frequencies in the context of the data.
♦ **ALG1-A.44.7:** Summarize categorical data for two categories in two-way frequency tables.
♦ **ALG1-A.44.8:** Analyze data from tables.

45. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. [S-ID6]

Objectives:

♦ **ALG1-A.45.1:** Define scatter plot.
ALG1-A.45.2: Describe how the variables on a scatter plot are related.
ALG1-A.45.3: Represent data of two quantitative variables on a scatter plot.
♦ **ALG1-A.45.4:** Construct a scatter plot using given data.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* [S-ID6a]

Objectives:

ALG1-A.45a.1: Use functions fitted to data to solve problems in the context of the data.
ALG1-A.45a.2: Fit function to the data.
ALG1-A.45a.3: Find the equation of an exponential function given data points.
ALG1-A.45a.4: Find the equation of a quadratic function given data points.
♦ **ALG1-A.45a.5:** Find the equation of a line given data points.
♦ **ALG1-A.45a.6:** Develop a table of values from data on a graph.

b. Informally assess the fit of a function by plotting and analyzing residuals. [S-ID6b]

Objectives:

♦ **ALG1-A.45b.1:** Define fit of a function and residuals.
ALG1-A.45b.2: Informally assess the fit of a function by plotting one set of points from the table of values.
ALG1-A.45b.3: Informally assess the fit by inputting an *x*-value from the table into the function and comparing the results to the value on the table.

c. Fit a linear function for a scatter plot that suggests a linear association. [S-ID6c]

Objectives:

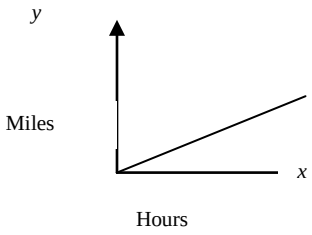
♦ **ALG1-A.45c.1:** Define linear function and scatter plot.
♦ **ALG1-A.45c.2:** Write the equation of a line given two points.
♦ **ALG1-A.45c.3:** Find the slope of a line given two points.

Interpret linear models.

46. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. [S-ID7]

Objectives:

ALG1-A.46.1: Define slope as a rate of change.
♦ **ALG1-A.46.2:** Understand that the *y*-intercept is the initial amount in the context of the data.
♦ **ALG1-A.46.3:** Understand that rate of change in the context of the data is the label of the *y-axis* divided by the label of the *x-axis*.
Example:



Rate of change is miles/hour

47. Compute (using technology) and interpret the correlation coefficient of a linear fit. [S-ID8]

Objectives:

♦ **ALG1-A.47.1:** Define correlation coefficient.
ALG1-A.47.2: Interpret the correlation coefficient of a linear fit.
♦ **ALG1-A.47.3:** Compute (using technology) the correlation coefficient of a linear fit.
ALG1-A.47.4: Input data into technology to find line of best fit.

48. Distinguish between correlation and causation. [S-ID9]

Objectives:
ALG1-A.48.1: Define correlation and causation.
ALG1-A.48.2: Distinguish between correlation and causation.

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.
(Link to data from simulations or experiments)

49. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [S-CP1]

Objectives:
♦ **ALG1-A.49.1:** Define subsets, sample space, outcomes, union, intersection, and complement.
ALG1-A.49.2: Describe events as subsets of a sample space (the set of outcomes).
ALG1-A.49.3: Describe events using characteristics (or categories) of the outcomes.
♦ **ALG1-A.49.4:** Describe events as unions of other events ("or").
♦ **ALG1-A.49.5:** Describe events as intersections of other events ("and").
♦ **ALG1-A.49.6:** Describe events as complements of other events ("not").

50. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. [S-CP2]

Objectives:
♦ **ALG1-A.50.1:** Define probability, ratio, simple event, compound event, and independent event.
ALG1-A.50.2: Determine the probability of a compound event.
♦ **ALG1-A.50.3:** Determine the probability of an independent event.
ALG1-A.50.4: Determine the probability of a simple event by expressing the probability as a ratio, percent, or decimal.
♦ **ALG1-A.50.5:** Identify the probability of an event that is certain as 1 or impossible as 0.
♦ **ALG1-A.50.6:** Solve word problems involving probability.
♦ **ALG1-A.50.7:** Use proportional relationships to solve multi-step ratio and percent problems.
ALG1-A.50.8: Recognize and represent proportional relationships as ratios between two quantities.

GEOMETRY

The Geometry course builds on Algebra I concepts and increases students' knowledge of shapes and their properties through geometry-based applications, many of which are observable in aspects of everyday life. This knowledge helps develop visual and spatial sense and strong reasoning skills. The Geometry course requires students to make conjectures and to use reasoning to validate or negate these conjectures. The use of proofs and constructions is a valuable tool that enhances reasoning skills and enables students to better understand more complex mathematical concepts. Technology should be used to enhance students' mathematical experience, not replace their reasoning abilities. Because of its importance, this Euclidean geometry course is required of all students receiving an Alabama High School Diploma.

School systems may offer Geometry and Geometry A and Geometry B.

Content standards 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 27, 31, 32, 33, 34, 35, 43, 44, 45, 46, 47, 48, 49, 50, and 51 must be taught in the Geometry A course.

Content standards 2, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 36, 37, 38, 39, 40, 41, and 42 must be taught in the Geometry B course.

Students will:

GEOMETRY

Congruence

Experiment with transformations in the plane.

1. Know the precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc. [G-CO1]

Objectives:

- ♦ **GEO-A.1.1:** Define angle, circle, perpendicular line, parallel line, line segment, and distance.
- **GEO-A.1.2:** Describe angle, circle, perpendicular line, parallel line, line segment, and distance.
- ♦ **GEO-A.1.3:** Identify angle, circle, perpendicular line, parallel line, line segment, and distance.
- **GEO-A.1.4:** Illustrate a point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). [G-CO2]

Objectives:

◊ **GEO-B.2.1:** Define distance, angle, input, output, plane, translation, and transformations.
GEO-B.2.2: Compare transformation that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
◊ **GEO-B.2.3:** Describe transformations as functions that take points in a plane as inputs and give other points as outputs.
GEO-B.2.4: Represent transformation in the plane.
◊ **GEO-B.2.5:** Generate an input output table.
◊ **GEO-B.2.6:** Measure distance and angle(s) of an image.

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. [G-CO3]

Objectives:

♦ **GEO-A.3.1:** Define a rectangle, parallelogram, trapezoid, regular polygon, rotation and reflection.
♦ **GEO-A.3.2:** Identify which reflections maintain the original properties of the shape.
♦ **GEO-A.3.3:** Demonstrate rotation and reflections on rectangle, parallelogram, trapezoid, regular polygon.
♦ **GEO-A.3.4:** Distinguish between a rotation and a reflection given an illustration.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. [G-CO4]

Objectives:

♦ **GEO-A.4.1:** Define rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
GEO-A.4.2: Discuss rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
GEO-A.4.3: Recognize the types of angles, circle, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. [G-CO5]

Objectives:

GEO-A.5.2: Describe the process of transforming a given figure.
♦ **GEO-A.5.3:** Illustrate figures transformed by a rotation, reflection or translation.
GEO-A.5.4: Describe the effects of rotations, reflection, and translations on two dimensional figures using coordinates.
♦ **GEO-A.5.5:** Graph a figure on a coordinate plane.
GEO-A.5.6: Plot points on a coordinate plane.

Understand congruence in terms of rigid motions. ***(Build on rigid motions as a familiar starting point for development of concept of geometric proof.)***

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. [G-CO6]

Objectives:

GEO-A.6.2: Applying the definition of congruence determine if two figures are congruent.

♦ **GEO-A.6.3:** Illustrate a vertical and horizontal shift on a coordinate plane.
Example: Rectangle *PQRS* has vertices *P(-3,5), Q(-4,2), R (3,0), 5(4,3).* Translate *PQRS* vertically 3 units.

GEO-A.6.4: Graph a figure on a coordinate plane.

GEO-A.6.5: Plot points on a coordinate plane.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. [G-CO7]

Objectives:

♦ **GEO-A.7.1:** Define congruent, corresponding, triangles, and angles.

GEO-A.7.2: Discuss the concept of *if and only if.*

♦ **GEO-A.7.3:** Compare angles and sides of two triangles to determine congruency**.**

♦ **GEO-A.7.4:** Identify corresponding parts of triangles.

♦ **GEO-A.7.5:** Measure sides and angles of triangles.

8. Explain how the criteria for triangle congruence, angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), follow from the definition of congruence in terms of rigid motions. [G-CO8]

Objectives:

GEO-A.8.1: Evaluate the properties of the triangles to prove congruency.

♦ **GEO-A.8.2:** Determine corresponding parts of triangles.

♦ **GEO-A.8.3:** State the angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), Theorems.

♦ **GEO-A.8.4:** Interpret specific symbols (hash marks and arcs) on shapes.

Prove geometric theorems. ***(Focus on validity of underlying reasoning while using variety of ways of writing proofs.)***

9. Prove theorems about lines and angles. *Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* [G-CO9]

Objectives:

♦ **GEO-A.9.1:** Define vertical angle, transversal, parallel lines, alternate interior angles, corresponding angles, perpendicular bisector, line segment, equidistant, and endpoints.
GEO-A.9.2: Develop a process that demonstrates the logical order of properties to form a proof.
GEO-A.9.3: Illustrate vertical angle, transversal, parallel lines, alternate interior angles, corresponding angles, perpendicular bisector, line segment, equidistant, endpoints, and their properties.
GEO-A.9.4: Arrange statements to form a logical order.
♦ **GEO-A.9.5:** Identify measures of vertical angle, alternate interior angles, and corresponding angles.

10. Prove theorems about triangles. *Theorems include measures of interior angles of a triangle sum to 180º, base angles of isosceles triangles are congruent, the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length, and the medians of a triangle meet at a point.* [G-CO10]

Objectives:

♦ **GEO-A.10.1:** Define interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, median, intersection.
GEO-A.10.2: Develop a process that demonstrates the logical order of properties to form a proof.
GEO-A.10.3: Illustrate interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, median, intersection, and their properties.
GEO-A.10.4: Arrange statements to form a logical order.
♦ **GEO-A.10.5:** Identify measures of interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, median.
♦ **GEO-A.10.6:** Find the measure of the third interior angle when given the measure of the other two interior angles.

11. Prove theorems about parallelograms. *Theorems include opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other; and conversely, rectangles are parallelograms with congruent diagonals.* [G-CO11]

Objectives:

♦ **GEO-A.11.1:** Define opposite sides, opposite angles, diagonals, parallelogram, bisector, and converse.
GEO-A.11.2: Develop a process that demonstrates the logical order of properties to form a proof.
GEO-A.11.3: Illustrate opposite sides, opposite angles, diagonals, parallelograms, bisectors and their properties.
GEO-A.11.4: Arrange statements to form a logical order.
♦ **GEO-A.11.5:** Identify measures of opposite sides, opposite angles, or diagonals of parallelograms.

Make geometric constructions. ***(Formalize and explain processes.)***

12. Make formal geometric constructions with a variety of tools and methods such as compass and straightedge, string, reflective devices, paper folding, and dynamic geometric software. *Constructions include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* [G-CO12]

Objectives:

◊ **GEO-B.12.1:** Construct a copy of a segment, copy of an angle, the bisection of a segment, the bisection of an angle, perpendicular line, perpendicular bisector of a line segment, and parallel lines.
GEO-B.12.2: Describe a specific construction process.
◊ **GEO-B.12.3:** Demonstrate the proper use of a geometric construction tools.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [G-CO13]

Objectives:

♦ **GEO-A.13.1:** Define equilateral triangle, regular hexagon, inscribed.
GEO-A.13.2: Apply the properties of equilateral triangle, regular hexagon, square and circle.
♦ **GEO-A.13.3:** Recognize given figures as inscribed.

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.

14. Verify experimentally the properties of dilations given by a center and a scale factor. [G-SRT1]

Objectives:

♦ **GEO-A.14.1:** Define dilation and scale factor.
GEO-A.14.2: Illustrate dilation.
♦ **GEO-A.14.3:** Apply a scale factor.

a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. [G-SRT1a]

Objectives:

GEO-A.14a.1: Illustrate when given an original figure with a line (e.g., *m*) through it, not through the center, a parallel line to *m* will be created when the dilation is performed.

Example: Given a line $x=\frac{1}{2}$, dilate the graph and line by 2. What happened to the line?

GEO-A.14a.2: Illustrate when given an original figure with a line (e.g., *m*) through its center the line will remain unchanged when the dilation is performed.
GEO-A.14a.3: Discuss the properties of parallel lines.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. [G-SRT1b]

Objectives:

GEO-A.14b.1: Determine the change in length of a line segment after dilation.
Example: Find the distance of line *AB*, given *A(0,0)* and *B(2,3)*, after dilating AB by a scale factor of °.

GEO-A.14b.2: Dilate a line segment.

15. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. [G-SRT2]

Objectives:

♦ **GEO-A.15.1:** Define similarity and proportions.
♦ **GEO-A.15.2:** Compare two figures in terms of similarity.
GEO-A.15.3: Create proportional equations from given information.
GEO-A.15.4: Solve proportional equations.
GEO-A.15.5: Demonstrate that equivalent ratios are proportions.

16. Use the properties of similarity transformations to establish the angle-angle (AA) criterion for two triangles to be similar. [G-SRT3]

Objectives:

GEO-A.16.1: Evaluate the AA postulate to prove similarity.
♦ **GEO-A.16.2:** Determine corresponding angles of triangles.
♦ **GEO-A.16.3:** State the AA postulate.
GEO-A.16.4: Interpret specific symbols (hash marks and arcs) on shapes.

Prove theorems involving similarity.

17. Prove theorems about triangles. *Theorems include a line parallel to one side of a triangle divides the other two proportionally, and conversely; and the Pythagorean Theorem proved using triangle similarity.* [G-SRT4]

Objectives:

◊ **GEO-B.17.1:** Define Pythagorean Theorem.
GEO-B.17.2: Develop a process that demonstrates the logical order of properties to form a proof.
◊ **GEO-B.17.3:** Use Pythagorean Theorem to find the missing side of a right triangle.
GEO-B.17.4: Illustrate a line parallel to one side of a triangle divides the other two proportionally, and conversely; and the Pythagorean Theorem proved using triangle similarity.
GEO-B.17.5: Arrange statement to form a logical order.
◊ **GEO-B.17.6:** Identify the parts of a right triangle.
Examples: legs, hypotenuse, right angle

18. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. [G-SRT5]

Objectives:

GEO-B.18.1: Develop an equation from given information to prove congruence or similarity.
◊ **GEO-B.18.2:** Illustrate congruence and similarity in geometric figures.
GEO-B.18.3: Apply proportional reasoning to real world scenarios.
◊ **GEO-B.18.4:** Solve proportions.

Define trigonometric ratios and solve problems involving right triangles.

19. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle leading to definitions of trigonometric ratios for acute angles. [G-SRT6]

Objectives:

◊ **GEO-B.19.1:** Define trigonometric ratios for acute angles**.**
GEO-B.19.2: Apply properties of similarity to demonstrate the trigonometric ratios of right triangles.
◊ **GEO-B.19.3:** Identify corresponding parts of similar triangles.

20. Explain and use the relationship between the sine and cosine of complementary angles. [G-SRT7]

Objectives:

◊ **GEO-B.20.1:** Define sine, cosine, and complementary angles.
GEO-B.20.2: Discuss the relationship between sine and cosine angles within a triangle.
◊ **GEO-B.20.3:** Find the complement of a given angle.

21. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* [G-SRT8]

Objectives:

◊ **GEO-B.21.1:** Create an equation using the given information**.**
◊ **GEO-B.21.2:** Solve equations involving exponents and radicals.
◊ **GEO-B.21.3:** Solve equations using ratios.
◊ **GEO-B.21.4:** Label parts of a right triangle.

Apply trigonometry to general triangles.

22. (+) Derive the formula $A = (\frac{1}{2})ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. [G-SRT9]

Objectives:

GEO-B.22.1: Prove the formula $A = (\frac{1}{2})ab \sin(C)$.

GEO-B.22.2: Discuss the formula $A = (\frac{1}{2})ab \sin(C)$.

GEO-B.22.3: Illustrate the formula $A = (\frac{1}{2})ab \sin(C)$.

23. (+) Prove the Law of Sines and the Law of Cosines and use them to solve problems. [G-SRT10]

Objectives:
GEO-B.23.1: Define the Law of Sines and the Law of Cosines**.**
GEO-B.23.2: Apply the Law of Sines and the Law of Cosines**.**

24. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). [G-SRT11]

Objectives:
GEO-B.24.1: Define the Law of Sines and the Law of Cosines**.**
GEO-B.24.2: Create an equation using the given information.
GEO-B.24.3: Solve real world problems using the Law of Sines and the Law of Cosines.

Circles

Understand and apply theorems about circles.

25. Prove that all circles are similar. [G-C1]

Objectives:
◊ **GEO-B.25.1:** Compare the parts of different circles thereby proving similarity.
◊ **GEO-B.25.2:** Identify parts of a circle.

26. Identify and describe *relationships* among inscribed angles, radii, and chords. Include the relationship *between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle*. [G-C2]

Objectives:
◊ **GEO-B.26.1:** Define inscribed, central, and circumscribed angles, radius, chord, tangent, and diameter.
GEO-B.26.2: Discuss the relationship among inscribed angles, radii, and chords.
◊ **GEO-B.26.3:** Illustrate radii, chords, diameters, tangents to curve, central, inscribed, and circumscribed angles.

27. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. [G-C3]

Objectives:
♦ **GEO-A.27.1:** Define inscribed and circumscribed circle of a triangle.
GEO-A.27.2: Develop a process that demonstrates the logical order of properties to form a proof.
GEO-A.27.3: Identify properties of angles from given information.
♦ **GEO-A.27.4:** Illustrate inscribed and circumscribed circles of a triangle and quadrilaterals inscribed in a circle.

28. (+) Construct a tangent line from a point outside a given circle to the circle. [G-C4]

Objectives:
GEO-B.28.1: Define tangent line.
GEO-B.28.2: Illustrate a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles. ***(Radian introduced only as unit of measure.)***

29. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. [G-C5]

Objectives:

◊ **GEO-B.29.1:** Define arc length, radian measure, and sector.

GEO-B.29.2: Prove the length of the arc intercepted by an angle is proportional to the radius by similarity.

GEO-B.29.3: Prove the formula for the area of the sector is $A = \frac{N}{360}\pi r^2$.

◊ **GEO-B.29.4:** Illustrate an arc of a circle by constructing the radii of a circle.

Expressing Geometric Properties With Equations

Translate between the geometric description and the equation for a conic section.

30. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. [G-GPE1]

Objectives:

GEO-B.30.1: Compare the equation of a circle at the origin and the radius in the first quadrant to the Pythagorean Theorem of the triangle formed inside the circle.

GEO-B.30.2: Recall the process of completing the square (Algebra 1).

GEO-B.30.3: Given an equation of a circle, determine the center and radius.

Use coordinates to prove simple geometric theorems algebraically. ***(Include distance formula; relate to Pythagorean Theorem.)***

31. Use coordinates to prove simple geometric theorems algebraically. [G-GPE4]
    Example: Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

Objectives:

GEO-A.31.1: Apply formulas, and properties of polygons, angles, and lines to draw conclusions from the given information.

♦ **GEO-A.31.2:** Illustrate polygons created by given coordinates on a coordinate plane.

GEO-A.31.3: Recall distance formula, circle formula, Pythagorean Theorem, midpoint, perpendicular and parallel lines, properties of polygons.

GEO-A.31.4: Graph points on a coordinate plane.

32. Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). [G-GPE5]

Objectives:

♦ **GEO-A.32.1:** Define slope, point slope formula, slope-intercept formula, standard form of a line, parallel lines, and perpendicular lines.
GEO-A.32.2: Demonstrate and explain algebraically how parallel lines have no common points.
GEO-A.32.3: Demonstrate and explain algebraically how perpendicular lines have only one common point.
GEO-A.32.4: Write and solve equations of parallel and perpendicular lines.
♦ **GEO-A.32.5:** Illustrate graphically how parallel lines have no common points.
♦ **GEO-A.32.6:** Illustrate graphically how perpendicular lines have only one common point.
♦ **GEO-A.32.7:** Find the slope of a given line.
GEO-A.32.8: Write an equation of a line.

33. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. [G-GPE6]

Objectives:

♦ **GEO-A.33.1:** Define line segment, distance formula, and ratio.
GEO-A.33.2: Set-up an equation to find the missing endpoint.
Example: Find the coordinates of the point $\frac{1}{3}$ the distance from point A on line segment AB with endpoints (1,5) and (4,8).
GEO-A.33.3: Apply properties of ratios.
♦ **GEO-A.33.4:** Solve equations using the distance formula.
GEO-A.33.5: Find a point on a number line according to a given ratio from a given point.

34. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* [G-GPE7]

Objectives:

♦ **GEO-A.34.1:** Define perimeter, polygons, areas of triangles, area of rectangles, distance formula.
GEO-A.34.2: Use the distance formula to find the lengths of sides of a given polygon.
♦ **GEO-A.34.3:** Apply the found lengths to the appropriate formulas for area and perimeter.
♦ **GEO-A.34.4:** Use a given illustration to determine area or perimeter**.**

Use coordinates to prove simple geometric theorems algebraically.

35. Determine areas and perimeters of regular polygons, including inscribed or circumscribed polygons, given the coordinates of vertices or other characteristics.

Objectives:

♦ **GEO-A.35.1:** Define area, perimeter, regular polygons, inscribed polygons, circumscribed polygons, and vertices.
GEO-A.35.2: Analyze the given information to develop a logical process to calculate area or perimeter.
GEO-A.35.3: Create equations for area and perimeter based on given information.
GEO-A.35.4: Recall properties of polygons.
♦ **GEO-A.35.5:** Illustrate graphically an inscribed or circumscribed polygon.
GEO-A.35.6: Solve given area and perimeter equations.
GEO-A.35.7: Plot given coordinates on the Cartesian plane.

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

36. Give an informal argument for the formulas for the circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* [G-GMD1]

Objectives:

◊ **GEO-B.36.1:** Define circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone, oblique, radius, diameter, and height, base.
GEO-B.36.2: Compare the characteristics and volume of oblique and right solids.
GEO-B.36.3: Identify the characteristics of solids.
◊ **GEO-B.36.4:** Calculate the circumference of a circle; area of a circle; and volumes of a cylinder, pyramid, and cone.
GEO-B.36.5: Illustrate a right and oblique solid.

37. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* [G-GMD3]

Objectives:

◊ **GEO-B.37.1:** Define cylinder, pyramid, cone, and sphere. Define the volume formulas of cylinders, pyramids, cones, and spheres.
GEO-B.37.2: Apply the appropriate volume formula for the given solid.
GEO-B.37.3: Identify the necessary characteristics of a given solid to solve for its volume**.**
◊ **GEO-B.37.4:** Calculate the volume of cylinders, pyramids, cones, and spheres.
◊ **GEO-B.37.5:** Calculate the area of the base shape**.**
◊ **GEO-B.37.6:** Identify the base shape.

38. Determine the relationship between surface areas of similar figures and volumes of similar figures.

Objectives:

GEO-B.38.1: Compare surface areas of similar figures and volumes of similar figures to determine a relationship**.**

GEO-B.38.2: Identify the necessary characteristics of a given solid to solve for its volume and surface area.

GEO-B.38.3: Calculate the surface area and the volume of figures.

GEO-B.38.4: Calculate the area of the base shape**.**

GEO-B.38.5: Identify the base shape.

Visualize relationships between two-dimensional and three-dimensional objects.

39. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. [G-GMD4]

Objectives:

◊ **GEO-B.39.1:** Define two-dimensional objects and three-dimensional objects**.**

GEO-B.39.2: Identify the two-dimensional figures that result from slicing three-dimensional figures as in plane section of right rectangular prisms and right rectangular pyramids.

GEO-B.39.3: Identify three-dimensional objects generated by rotations of two-dimensional objects (as in rotating a circle to create a sphere).

◊ **GEO-B.39.4:** Distinguish between two-dimensional and three dimensional objects.

Modeling With Geometry

Apply geometric concepts in modeling situations.

40. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* [G-MG1]

Objectives:

GEO-B.40.1: Estimate the dimensions of a given object.

GEO-B.40.2: Discuss the properties of a given object.

◊ **GEO-B.40.3:** Identify the relationship of geometric representations to real-life objects.

41. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, British Thermal Units (BTUs) per cubic foot).* [G-MG2]

Objectives:

◊ **GEO-B.41.1:** Define density, area, volume.

GEO-B.41.2: Discuss the relationship between units in each modeling situation.

GEO-B.41.3: Recognize appropriate units for various situations.

◊ **GEO-B.41.4:** Illustrate a design conflict (e.g., draw a chair and a desk where the chair will not fit under the desk.)

42. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios).* [G-MG3]

Objectives:
GEO-B.42.1: Demonstrate deductive reasoning skills.
GEO-B.42.2: Dissect the information in a given problem.
GEO-B.42.3: Recall geometric formulas and methods.
GEO-B.42.4: Apply the appropriate geometric formulas and methods.

STATISTICS AND PROBABILITY

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data. ***(Link to data from simulations or experiments.)***

43. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. [S-CP3]

Objectives:
♦ **GEO-A.43.1:** Define conditional probability and independence.
GEO-A.43.2: Analyze independence of a given set.
GEO-A.43.3: Recognize the equation *P*(*A* and *B*)/*P*(*B*) as the conditional probability of *A* given *B*.
GEO-A.43.4: Apply independence in conditional probability problems.
♦ **GEO-A.43.5:** Interpret a Venn Diagram.
♦ **GEO-A.43.6:** Draw a Venn Diagram.

44. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP4]
    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

Objectives:
♦ **GEO-A.44.1:** Define two-way frequency table, sample space, independence, and conditional probability.
GEO-A.44.2: Interpret two-way frequency tables of data.
GEO-A.44.3: Construct a two-way frequency table of data.
GEO-A.44.4: Approximate conditional probabilities based on a two-way frequency table.
GEO-A.44.5: Determine independence of events.
♦ **GEO-A.44.6:** Collect data for construction of frequency tables.

45. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP5]
    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Objectives:

♦ **GEO-A.45.1:** Define conditional probability and independence.
GEO-A.45.2: Discuss conditional probability and independence in everyday language.
GEO-A.45.3: Apply conditional probability and independence in everyday situations.
♦ **GEO-A.45.4:** Match dependent and independent events to given everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

46. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. [S-CP6]

Objectives:

♦ **GEO-A.46.1:** Define conditional probability, Venn Diagram, and outcomes.
GEO-A.46.2: Determine the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A.*
GEO-A.46.3: Analyze the results of a given model.
GEO-A.46.4: Create a model of a given conditional probability.
GEO-A.46.5: Interpret a Venn Diagram.
GEO-A.46.6: Create a Venn Diagram.

47. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. [S-CP7]

Objectives:

GEO-A.47.1: Define Addition Rule for Probability.
GEO-A.47.2: Compute the probability of *A* or *B* using the Addition Rule.
GEO-A.47.3: Identify the necessary information from a model to accurately apply the Addition Rule.
GEO-A.47.4: Determine the probability of conditional events.
♦ **GEO-A.47.5:** Determine the probability of independent events.

48. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. [S-CP8]

Objectives:

GEO-A.48.1: Define Multiplication Rule of Probability and uniform probability.
GEO-A.48.2: Compute the probability using the Multiplication Rule.
GEO-A.48.3: Identify the necessary information from a model to accurately apply the Multiplication Rule.
GEO-A.48.4: Recognize the meaning of symbols in the Multiplication Rule.
GEO-A.48.5: Recall the associative property of multiplication.

49. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. [S-CP9]

Objectives:
GEO-A.49.1: Define permutation, combination, factorial, and compound events.
GEO-A.49.2: Solve probabilities of compound events using permutations or combinations.
GEO-A.49.3: Calculate problems containing factorials.

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. ***(Introductory; apply counting rules.)***

50. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

Objectives:
GEO-A.50.1: Describe processes that can be used to make fair decisions.
♦ **GEO-A.50.2:** Demonstrate a random number generator.
♦ **GEO-A.50.3:** Generate a table/chart of outcomes.

51. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Objectives:
GEO-A.51.1: Examine the context of given probabilities.
GEO-A.51.2: Interpret decisions and strategies of given probability concepts.
GEO-A.51.3: Interpret expressions.

ALGEBRAIC CONNECTIONS

Algebraic Connections is a course designed for students who wish to increase their mathematical knowledge and skills prior to enrollment in the Algebra II course or the Algebra II With Trigonometry course. Algebraic Connections expands upon the concepts of Algebra I and Geometry, with an emphasis on application-based problems. This course provides opportunities to incorporate the use of technology through its emphasis on applying functions to make predictions and to calculate outcomes. The prerequisites for Algebraic Connections are Algebra I and Geometry.

Students will:

ALGEBRA

Modeling

1. Create algebraic models for application-based problems by developing and solving equations and inequalities, including those involving direct, inverse, and joint variation.
   Example: The amount of sales tax on a new car is directly proportional to the purchase price of the car. If the sales tax on a $20,500 car is $1,600, what is the purchase price of a new car that has a sales tax of $3,200?
   Answer: The purchase price of the new car is $41,000.

Objectives:

AC.1.1: Define algebraic model, application-based problem, proportion, cross product, inequality, direct variation, inverse variation, and joint variation.
AC.1.2: Solve inverse and joint variation equations for the constant of proportionality.
AC.1.3: Solve a direct variation equation for the constant of proportionality.
AC.1.4: Solve for a missing value of a proportion.
AC.1.5: Solve two-step equations using addition, subtraction, multiplication, and division.
AC.1.6: Solve one-step equations using multiplication and division.

2. Solve application-based problems by developing and solving systems of linear equations and inequalities.

Objectives:

AC.2.1: Solve a system of linear equations using the substitution method**.**
AC.2.2: Solve a system of linear equations using the elimination method.
AC.2.3: Write a system of linear equations using given information.
AC.2.4: Graph a system of linear equations and find the point of intersection.
   Example: Solve the following system by graphing:
   $y = \frac{1}{2} x + 5$
   $y = 3x - 2$
AC.2.5: Graph a linear equation.

3. Use formulas or equations of functions to calculate outcomes of exponential growth or decay.
   Example: Solve problems involving compound interest, bacterial growth, carbon-14 dating, and depreciation.

Objectives:
AC.3.1: Define exponential growth, exponential decay, compound interest, and half-life problems.
AC.3.2: Solve depreciation problems.
Example: John bought a car for $20,000. After five years the car's value is $6000.00. Find the rate of depreciation.
AC.3.3: Identify components of the continuously compounded interest formula.
AC.3.4: Identify components of the half-life formula.
AC.3.5: Understand rate of change as it relates to growth or decay.
Example: Given a 34% rate of change, find b and determine if b represents growth or decay.
Solution: b = 1 + r
b = 1+.34
b = 1.34; growth factor
AC.3.6: Identify components of the exponential growth and decay formula.
Example: Given y = ab^x; state the meanings of the variables a, b ,x ,y
Solution: y = amount of substance remaining after x time units
x = elapsed time
a = initial amount
b = growth or decay factor

Graphing

4. Determine maximum and minimum values of a function using linear programming procedures.
   Example: Observe the boundaries $x \geq 0$, $y \geq 0$, $2x - 3y + 15 \geq 0$, and $x \leq 9$ to find the maximum and minimum values of $f(x, y) = 3x + 5y$.

Objectives:
AC.4.1: Define linear programming.
AC.4.2: Solve systems of line inequalities.
AC.4.3: Graph linear functions and find their intersection points.

5. Determine approximate rates of change of nonlinear relationships from graphical and numerical data.

Objectives:
AC.5.1: Create a scatter plot and line of best fit using data from a spreadsheet.
AC.5.2: Organize numerical data in a spreadsheet.

a. Create graphical representations from tables, equations, or classroom-generated data to model consumer costs and to predict future outcomes.

Objectives:
AC.5a.1: Create graphical representations from classroom-generated data to model consumer costs.
AC.5a.2: Create graphical representations from classroom-generated data to predict future outcomes.
AC.5a.3: Create graphical representations from equations to model consumer costs.
AC.5a.4: Create graphical representations from equations to predict future outcomes.
AC.5a.5: Create graphical representations from tables to model consumer costs.
AC.5a.6: Create graphical representations from tables to predict future outcomes.

6. Use the extreme value of a given quadratic function to solve applied problems.
   Example: Determine the selling price needed to maximize profit.

Objectives:
AC.6.1: Define quadratic function, extreme value, maximum and minimum.
AC.6.2: Solve applied problems involving maximum and minimum values.
AC.6.3: Find the maximum or minimum value of a quadratic function.

Finance

7. Use analytical, numerical, and graphical methods to make financial and economic decisions, including those involving banking and investments, insurance, personal budgets, credit purchases, recreation, and deceptive and fraudulent pricing and advertising.
   Examples: Determine the best choice of certificates of deposit, savings accounts, checking accounts, or loans. Compare the costs of fixed- or variable-rate mortgage loans. Compare costs associated with various credit cards. Determine the best cellular telephone plan for a budget.

Objectives:
AC.7.1: Define certificate of deposit, fixed- or variable-rate mortgage loans.
AC.7.2: Create charts and graphs using spreadsheet software.

   a. Create, manually or with technological tools, graphs and tables related to personal finance and economics.
      Example: Use spreadsheets to create an amortization table for a mortgage loan or a circle graph for a personal budget.

Objectives:
AC.7a.1: Create manually graphs and tables related to personal finance.
AC.7a.2: Create manually graphs and tables related to economics.
AC.7a.3: Create using technological tools graphs and tables related to personal finance.
AC.7a.4: Create using technological tools graphs and tables related to economics.

GEOMETRY

Modeling

8. Determine missing information in an application-based situation using properties of right triangles, including trigonometric ratios and the Pythagorean Theorem.
   Example: Use a construction or landscape problem to apply trigonometric ratios and the Pythagorean Theorem.

Objectives:
AC.8.1: Define the trigonometric ratios: sine β, cosine β, and tangent β.
AC.8.2: Use the Pythagorean Theorem to compute the missing side of a right triangle.

Symmetry

9. Analyze aesthetics of physical models for line symmetry, rotational symmetry, or the golden ratio.
   Example: Identify the symmetry found in nature, art, or architecture.

Objectives:
AC.9.1: Define line symmetry, rotational symmetry, and the golden ratio.
AC.9.2: Locate models from art and architecture that demonstrate the golden ratio.
AC.9.3: Identify line symmetry and rotational symmetry in physical models.

Measurement

10. Critique measurements in terms of precision, accuracy, and approximate error.
    Example: Determine whether one candidate has a significant lead over another candidate when given their current standings in a poll and the margin of error.

Objectives:
AC.10.1: Analyze a political poll for precision and accuracy.
AC.10.2: Compare the poll to a standardized poll.

11. Use ratios of perimeters, areas, and volumes of similar figures to solve applied problems.
    Example: Use a blueprint or scale drawing of a house to determine the amount of carpet to be purchased.

Objectives:
AC.11.1: Define perimeter, area, volume and similar figures.
AC.11.2: Determine the relationship of the scale factor to perimeter, area, and volume.
AC.11.3: Find the scale factor of two similar figures.

STATISTICS AND PROBABILITY

Graphing

12. Create a model of a set of data by estimating the equation of a curve of best fit from tables of values or scatter plots.
    Examples: Create models of election results as a function of population change, inflation or employment rate as a function of time, cholesterol density as a function of age or weight of a person.

Objectives:
AC.12.1: Define curve of best fit, estimation, and model.
AC.12.2: Identify the curve of best fit from a table.
AC.12.3: Determine the effect of outliers on the data.

a. Predict probabilities given a frequency distribution.

Objectives:
AC.12a.1: Define frequency distribution.
AC.12a.2: Predict probabilities given a frequency distribution.

ALGEBRA II

Algebra II is a terminating course designed to extend students' algebraic knowledge and skills beyond Algebra I. Students are encouraged to solve problems using a variety of methods that promote the development of improved communication skills and foster a deeper understanding of mathematics. To help students appreciate the power of algebra, application-based problems are incorporated throughout the course. The use of appropriate technology is also encouraged for numerical and graphical investigations.

In contrast to the Algebra II With Trigonometry course, Algebra II does not meet the graduation requirements for the Alabama High School Diploma with Advanced Academic Endorsement due to the fact that it does not contain trigonometry content. Algebra II With Trigonometry or Algebra II is required to complete the graduation requirements for the Alabama High School Diploma. This course does not provide sufficient background to prepare students to pursue higher-level mathematics courses. The prerequisites for Algebra II are Algebra I and Geometry.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

Objectives:
ALG2.1.1: Define complex number and imaginary number.
ALG2.1.2: Demonstrate that any complex number can be written in terms of i.
ALG2.1.3: Identify the real and imaginary terms of a complex number.
ALG2.1.4: Review laws of integers.

2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

Objectives:
ALG2.2.1: Apply commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
ALG2.2.2: Review commutative, associative, and distributive properties.

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

3. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

Objectives:
ALG2.3.1: Solve quadratic equations with real coefficients that have complex solutions.
ALG2.3.2: Solve quadratic equations with real coefficients that have simple solutions.
ALG2.3.3: Review quadratic formula, completing the square, and factoring.
ALG2.3.4: Review the zero-product property.

4. (+) Extend polynomial identities to the complex numbers. [N-CN8]
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.

Objectives:
ALG2.4.1: Define conjugate pairs.
ALG2.4.2: Apply quadratic formula, completing the square, and factoring to complex numbers.
ALG2.4.3: Review complex numbers.

5. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

Objectives:
ALG2.5.1: Define the Fundamental Theorem of Algebra.
ALG2.5.2: Demonstrate the Fundamental Theorem of Algebra on quadratic polynomials.

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Polynomial and rational.)*

6. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]

Objectives:
ALG2.6.1: Define polynomial and rational number.
ALG2.6.2: Classify a polynomial (e.g. as a monomial, binomial, trinomial).

   a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]

Objectives:
ALG2.6a.1: Define terms, factors, and coefficients.
ALG2.6a.2: Identify terms in polynomial and rational expressions.
ALG2.6a.3: Identify factors in polynomial and rational expressions.
ALG2.6a.4: Identify coefficients in polynomial and rational expressions.
ALG2.6a.5: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient).

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
   Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Objectives:
ALG2.6b.1: Recognize one or more parts of a polynomial expression as a single entity.
ALG2.6b.2: Recognize one or more parts of a rational expression as a single entity.

7. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
   Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Objectives:
ALG2.7.1: Rewrite a polynomial expression in an alternative way.
ALG2.7.2: Rewrite a rational expression in an alternative way.
ALG2.7.3: Understand that rewriting an expression in different forms in a problem context can shed light on the problem.

Write expressions in equivalent forms to solve problems.

8. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
   Example: Calculate mortgage payments.

Objectives:
ALG2.8.1: Convert an expression to an alternative format.
ALG2.8.2: Recognize the best format for a specific application.
ALG2.8.3: Match equivalent expressions written in different formats.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. ***(Beyond quadratic.)***

9. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Objectives:
ALG2.9.1: Define term, monomial, binomial, trinomial, and polynomial.
ALG2.9.2: Multiply polynomial expressions.
ALG2.9.3: Add and subtract polynomial expressions.
ALG2.9.4: Use order of operations to evaluate and simplify algebraic and numerical expressions.
ALG2.9.5: Identify the terms in a polynomial expression.
ALG2.9.6: Explain the distributive property.

Understand the relationship between zeros and factors of polynomials.

10. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. [A-APR2]

Objectives:
ALG2.10.1: Show that long division and synthetic division of a polynomial function results in the same answer, which is also the value of the function.
ALG2.10.2: Review long division.
ALG2.10.3: Review synthetic division.
ALG2.10.4: Find the value of a function *f(x)*.

11. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Objectives:
ALG2.11.1: Set equations equal to zero and solve for x.
ALG2.11.2: Discuss common relations and anticipate their graphical appearance based on degree of a function.
ALG2.11.3: Use zeros to graph a function.
ALG2.11.4: Review completing the square, factoring, and quadratic formula.
ALG2.11.5: Review the zero-product property.

Use polynomial identities to solve problems.

12. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Objectives:
ALG2.12.1: Define Pythagorean triples and polynomial identities.
ALG2.12.2: Identify the polynomial identities used to manipulate numerical relationships.

13. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) [A-APR5]

Objectives:
ALG2.13.1: Define Binomial Theorem.
ALG2.13.2: Use Pascal's Triangle.
ALG2.13.3: Use Binomial Theorem.
ALG2.13.4: Solve factorial and combinatorial problems.
ALG2.13.5: Draw Pascal's Triangle.
ALG2.13.6: Recall rules of exponents.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

14. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

Objectives:
ALG2.14.1: Rewrite $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using long division.
ALG2.14.2: Rewrite $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection.
ALG2.14.3: Rewrite $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using a computer algebra system.
ALG2.14.4: Review long division of polynomial functions.
ALG2.14.5: Review writing remainders as fractions.

15. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. [A-APR7]

Objectives:
ALG2.15.1: Add, subtract, multiply, and divide rational expressions.
ALG2.15.2: Apply properties of multiplication and addition for rational numbers.

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

16. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

Objectives:
ALG2.16.1: Define equation, expression, variable, equality, and inequality.
ALG2.16.2: Create inequalities with one variable.
ALG2.16.3: Create equations with one variable.
ALG2.16.4: Solve two-step equations and inequalities.
ALG2.16.5: Solve one-step equations and inequalities.
ALG2.16.6: Compare and contrast equations and inequalities.
ALG2.16.7: Recognize inequality symbols including $<$, $>$, $\leq$, and $\geq$.

17. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

Objectives:
ALG2.17.1: Define ordered pair and coordinate plane.
ALG2.17.2: Create equations with two variables (exponential, quadratic and linear).
ALG2.17.3: Graph equations on coordinate axes with labels and scales (exponential, quadratic, and linear).
ALG2.17.4: Identify an ordered pair and plot it on the coordinate plane.

18. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

Objectives:
ALG2.18.1: Define systems of equations and constraints.
ALG2.18.2: Create a system of equations or inequalities to represent the given constraints.
ALG2.18.3: Create an equation or inequality to represent the given constraints.
ALG2.18.4: Determine if a solution to a system of equations or inequalities is viable.
ALG2.18.5: Determine if there is one solution, infinite solutions, or no solutions to a system of equations or inequalities.

19. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Objectives:
ALG2.19.1: Solve a familiar literal equation for a specific variable.
ALG2.19.2: Identify and isolate a specific variable.
ALG2.19.3: Demonstrate the process of solving multi-step equations.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning. ***(Simple rational and radical.)***

20. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Objectives:
ALG2.20.1: Define rational, irrational, and radical expressions and extraneous solutions.
ALG2.20.2: Simplify rational and radical equations.
ALG2.20.3: Apply properties of exponentials.
ALG2.20.4: Evaluate solutions by substituting into the original equation.

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

21. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Objectives:
ALG2.21.1: Define function, function notation, linear, polynomial, rational, radical, absolute value, exponential, and logarithmic functions, and transitive property.
ALG2.21.2: Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations.
ALG2.21.3: Find solutions to the equations $y = f(x)$ and $y = g(x)$ using the graphing calculator.
ALG2.21.4: Solve equations for y.
ALG2.21.5: Apply the properties of multiplicative inverses.
ALG2.21.6: Apply the properties of exponentials.
ALG2.21.7: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations.

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

22. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** [F-IF4]

Objectives:
ALG2.22.1: Define intercepts, intervals, relative maxima, relative minima, symmetry, end behavior, and periodicity.
ALG2.22.2: For a function that models a relationship between two quantities, find the periodicity.
ALG2.22.3: For a function that models a relationship between two quantities, find the end behavior.
ALG2.22.4: For a function that models a relationship between two quantities, find the symmetry.
ALG2.22.5: For a function that models a relationship between two quantities, find the intervals where the function is increasing, decreasing, positive, or negative.
ALG2.22.6: For a function that models a relationship between two quantities, find the relative maxima and minima.
ALG2.22.7: For a function that models a relationship between two quantities, find the x and y intercepts.

23. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Objectives:
ALG2.23.1: Define domain, range, relation, function, table of values, and mappings.
ALG2.23.2: Determine the appropriate domain for a given function.
ALG2.23.3: Identify functions from information in tables, sets of ordered pairs, and mappings.
ALG2.23.4: Translate verbal phrases into a function.
ALG2.23.5: Graph a function on a coordinate plane.
ALG2.23.6: Arrange data given as ordered pairs into a table and a table of values into ordered pairs.
ALG2.23.7: Identify the x and y values in an ordered pair.

24. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Objectives:
ALG2.24.1: Define average rate of change as slope.
ALG2.24.2: Estimate the rate of change from a graph (rise/run).
ALG2.24.3: Interpret the average rate of change.
ALG2.24.4: Calculate the average rate of change.
ALG2.24.5: Compute the slope of a line given two ordered pairs.
ALG2.24.6: Identify the slope, given slope-intercept form.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

25. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]

Objectives:
ALG2.25.1: Define piecewise-defined functions and step functions.
ALG2.25.2: Graph functions expressed symbolically by hand in simple cases.
ALG2.25.3: Graph functions expressed symbolically using technology for more complicated cases.

a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]

Objectives:
ALG2.25a.1: Define square root, cube root, and absolute value function.
ALG2.25a.2: Graph piecewise-defined functions.
ALG2.25a.3: Graph step functions.
ALG2.25a.4: Graph cube root functions.
ALG2.25a.5: Graph square root functions.
ALG2.25a.6: Graph absolute value functions.

b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]

Objectives:
ALG2.25b.1: Solve polynomial function for their zeros.
ALG2.25b.2: Plot the zeros on a coordinate plane.
ALG2.25b.3: Illustrate end behavior on graph.
ALG2.25b.4: Review multiplicity of zeros.

c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

Objectives:
ALG2.25c.1: Define exponential function, logarithmic function, trigonometric function, intercepts, end behavior, period, midline, and amplitude.
ALG2.25c.2: Graph logarithmic functions showing intercepts and end behavior.
ALG2.25c.3: Graph exponential functions showing intercepts and end behavior.
ALG2.25c.4: Graph trigonometric functions showing period, midline, and amplitude.

26. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

Objectives:
ALG2.26.1: Write a function defined by an expression in different but equivalent forms to reveal different properties of the function.
ALG2.26.2: Write a function defined by an expression in different but equivalent forms to explain different properties of the function.

27. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Objectives:
ALG2.27.1: Compare properties of two functions each represented in a different way.
ALG2.27.2: Identify properties of functions defined algebraically.
ALG2.27.3: Identify properties of functions defined by verbal description.
ALG2.27.4: Identify properties of functions defined graphically.
ALG2.27.5: Identify properties of functions defined numerically in tables.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

28. Combine standard function types using arithmetic operations. [F-BF1b]
    Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Objectives:
ALG2.28.1: Define standard function types as exponential, quadratic, and linear.
ALG2.28.2: Combine standard functions by dividing a constant function.
ALG2.28.3: Combine standard functions by multiplying a constant function.
ALG2.28.4: Combine standard functions by subtracting a constant function.
ALG2.28.5: Combine standard functions by adding a constant function.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

29. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

Objectives:
ALG2.29.1: Recognize even and odd functions from algebraic expressions for them.
ALG2.29.2: Recognize even and odd functions from their graphs.
ALG2.29.3: Experiment with various cases of functions and illustrate an explanation of the effects on the graph using technology. (Exponential, Quadratic, Absolute Value).
ALG2.29.4: Find the value of k given the graphs of $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).
ALG2.29.5: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

30. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
    Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Objectives:
ALG2.30.1: Solve an equation of the form $f(x) = c$ for a simple linear function f that has an inverse.
ALG2.30.2: Write an expression for the inverse of a simple linear function f of the form $f(x) = c$.
ALG2.30.3: Apply the substitution principle.
ALG2.30.4: Solve a multi-step equation.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. *(Logarithms as solutions for exponentials.)*

31. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

Objectives:
ALG2.31.1: Define logarithmic and exponential function.
ALG2.31.2: Recognize the inverse relationship of logarithmic function and exponential functions.
ALG2.31.3: Calculate the change of base formula for logarithms.
ALG2.31.4: Recall laws of exponents.
ALG2.31.5: Apply the properties of logarithms.
ALG2.31.6: Discuss the appropriateness of the solution.

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

32. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. [S-ID4]

Objectives:
ALG2.32.1: Define mean, standard deviation, normal distribution, normal curve, data sets, and population percentages.
ALG2.32.2: Calculate the estimated area under the curve.
ALG2.32.3: Estimate the population percentages.
ALG2.32.4: Calculate the standard deviation of the data set.
ALG2.32.5: Draw a normal curve using given data.

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

33. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. [S-IC1]

Objectives:
ALG2.33.1: Define statistics, random sample, population and parameter.
ALG2.33.2: Discuss statistics as a process for making inferences about population parameters based on a random sample from that population.
ALG2.33.3: Apply statistics as a process for making inferences about population parameters based on a random sample from that population.

34. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. [S-IC2]
    Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Objectives:
ALG2.34.1: Compare and contrast theoretical models and experimental models.
ALG2.34.2: Analyze the results.
ALG2.34.3: Perform the experiment manually, abstractly or using technology.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

35. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. [S-IC3]

Objectives:
ALG2.35.1: Define sample surveys, experiments, observational studies and randomization.
ALG2.35.2: Discuss how randomization relates to sample surveys, experiments and observational studies.
ALG2.35.3: Compare and contrast sample surveys, experiments and observational studies.

36. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. [S-IC4]

Objectives:
ALG2.36.1: Define population, mean, proportion, margin of error, simulation and random sampling.
ALG2.36.2: Use data from a sample survey to estimate a population mean or proportion
ALG2.36.3: Calculate an estimate of the population mean or proportion.
ALG2.36.4: Calculate a margin of error through the use of simulation models for random sampling.
ALG2.36.5: Simulate a model.

37. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. [S-IC5]

Objectives:
ALG2.37.1: Define randomized experiment, treatments, and parameters.
ALG2.37.2: Interpret the data.
ALG2.37.3: Simulate the experiment.
ALG2.37.4: Compare the treatments of the experiment.

38. Evaluate reports based on data. [S-IC6]

Objectives:
ALG2.38.1: Analyze the data.
ALG2.38.2: Determine whether the report accurately portrays the data.

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. ***(Include more complex situations.)***

39. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

Objectives:
ALG2.39.1: Describe processes that can be used to make fair decisions.
ALG2.39.2: Demonstrate a random number generator.
ALG2.39.3: Generate a table/chart of outcomes.

40. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Objectives:
ALG2.40.1: Examine the context of given probabilities.
ALG2.40.2: Interpret decisions and strategies of given probability concepts.
ALG2.40.3: Interpret expressions.

ALGEBRA II WITH TRIGONOMETRY

Algebra II With Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II With Trigonometry is required for all students pursuing the Alabama High School Diploma with Advanced Academic Endorsement. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II With Trigonometry course.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

Objectives:
ALG2T.1.1: Define complex number and imaginary number.
ALG2T.1.2: Demonstrate that any complex number can be written in terms of i.
ALG2T.1.3: Identify the real and imaginary terms of a complex number.
ALG2T.1.4: Review laws of integers.

2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

Objectives:
ALG2T.2.1: Apply commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
ALG2T.2.2: Review commutative, associative, and distributive properties.

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

3. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

Objectives:
ALG2T.3.1: Solve quadratic equations with real coefficients that have complex solutions.
ALG2T.3.2: Solve quadratic equations with real coefficients that have simple solutions.
ALG2T.3.3: Review quadratic formula, completing the square, and factoring.
ALG2T.3.4: Review the zero-product property.

4. (+) Extend polynomial identities to the complex numbers.
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. [N-CN8]

Objectives:
ALG2T.4.1: Define conjugate pairs.
ALG2T.4.2: Apply quadratic formula, completing the square, and factoring to complex numbers.
ALG2T.4.3: Review complex numbers.

5. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

Objectives:
ALG2T.5.1: Define the Fundamental Theorem of Algebra.
ALG2T.5.2: Demonstrate the Fundamental Theorem of Algebra on quadratic polynomials.

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Polynomial and rational.)*

6. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]

Objectives:
ALG2T.6.1: Define polynomial and rational number.

ALG2T.6.2: Classify a polynomial (e.g. as a monomial, binomial, trinomial).

   a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]

Objectives:
ALG2T.6a.1: Define terms, factors, and coefficients.
ALG2T.6a.2: Identify terms in polynomial and rational expressions.
ALG2T.6a.3: Identify factors in polynomial and rational expressions.
ALG2T.6a.4: Identify coefficients in polynomial and rational expressions.
ALG2T.6a.5: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient).

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
   Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Objectives:
ALG2T.6b.1: Recognize one or more parts of a polynomial expression as a single entity.
ALG2T.6b.2: Recognize one or more parts of a rational expression as a single entity.

7. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
   Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Objectives:
ALG2T.7.1: Rewrite a polynomial expression in an alternative way.
ALG2T.7.2: Rewrite a rational expression in an alternative way.
ALG2T.7.3: Understand that rewriting an expression in different forms in a problem context can shed light on the problem.

Write expressions in equivalent forms to solve problems.

8. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
   Example: Calculate mortgage payments.

Objectives:
ALG2T.8.1: Convert an expression to an alternative format.
ALG2T.8.2: Recognize the best format for a specific application.
ALG2T.8.3: Match equivalent expressions written in different formats.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. ***(Beyond quadratic.)***

9. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Objectives:
ALG2T.9.1: Define term, monomial, binomial, trinomial, and polynomial.
ALG2T.9.2: Multiply polynomial expressions.
ALG2T.9.3: Add and subtract polynomial expressions.
ALG2T.9.4: Use order of operations to evaluate and simplify algebraic and numerical expressions.
ALG2T.9.5: Identify the terms in a polynomial expression.
ALG2T.9.6: Explain the distributive property.

Understand the relationship between zeros and factors of polynomials.

10. (+) Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. [A-APR2]

Objectives:
ALG2T.10.1: Show that long division and synthetic division of a polynomial function results in the same answer, which is also the value of the function.
ALG2T.10.2: Review long division.
ALG2T.10.3: Review synthetic division.
ALG2T.10.4: Find the value of a function *f(x).*

11. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Objectives:
ALG2T.11.1: Set equations equal to zero and solve for x.
ALG2T.11.2: Discuss common relations and anticipate their graphical appearance based on degree of a function.
ALG2T.11.3: Use zeros to graph a function.
ALG2T.11.4: Review completing the square, factoring, and quadratic formula.
ALG2T.11.5: Review the zero-product property.

Use polynomial identities to solve problems.

12. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

ALG2T.12.1: Define Pythagorean triples and polynomial identities.
ALG2T.12.2: Identify the polynomial identities used to manipulate numerical relationships.

13. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) [A-APR5]

Objectives:
ALG2T.13.1: Define Binomial Theorem.
ALG2T.13.2: Use Pascal's Triangle.
ALG2T.13.3: Use Binomial Theorem.
ALG2T.13.4: Solve factorial and combinatorial problems.
ALG2T.13.5: Draw Pascal's Triangle.
ALG2T.13.6: Recall rules of exponents.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

14. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

ALG2T.14.1: Rewrite $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using long division.
ALG2T.14.2: Rewrite $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection.
ALG2T.14.3: Rewrite $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using a computer algebra system.
ALG2T.14.4: Review long division of polynomial functions.
ALG2T.14.5: Review writing remainders as fractions.

15. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. [A-APR7]

ALG2T.15.1: Add, subtract, multiply, and divide rational expressions.
ALG2T.15.2: Apply properties of multiplication and addition for rational numbers.

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

16. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

Objectives:
ALG2T.16.1: Define equation, expression, variable, equality, and inequality.
ALG2T.16.2: Create inequalities with one variable.
ALG2T.16.3: Create equations with one variable.
ALG2T.16.4: Solve two-step equations and inequalities.
ALG2T.16.5: Solve one-step equations and inequalities.
ALG2T.16.6: Compare and contrast equations and inequalities.
ALG2T.16.7: Recognize inequality symbols including $<$, $>$, $\leq$, and $\geq$.

17. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

ALG2T.17.1: Define ordered pair and coordinate plane.
ALG2T.17.2: Create equations with two variables (exponential, quadratic and linear).
ALG2T.17.3: Graph equations on coordinate axes with labels and scales (exponential, quadratic, and linear).
ALG2T.17.4: Identify an ordered pair and plot it on the coordinate plane.

18. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

ALG2T.18.1: Define systems of equations and constraints.
ALG2T.18.2: Create a system of equations or inequalities to represent the given constraints.
ALG2T.18.3: Create an equation or inequality to represent the given constraints.
ALG2T.18.4: Determine if a solution to a system of equations or inequalities is viable.
ALG2T.18.5: Determine if there is one solution, infinite solutions, or no solutions to a system of equations or inequalities.

19. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

ALG2T.19.1: Solve a familiar literal equation for a specific variable.
ALG2T.19.2: Identify and isolate a specific variable.
ALG2T.19.3: Demonstrate the process of solving multi-step equations.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning, and explain the reasoning. ***(Simple rational and radical)***

20. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Objectives:
ALG2T.20.1: Define rational, irrational, and radical expressions and extraneous solutions.
ALG2T.20.2: Simplify rational and radical equations.
ALG2T.20.3: Apply properties of exponentials.
ALG2T.20.4: Evaluate solutions by substituting into the original equation.

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

21. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Objectives:
ALG2T.21.1: Define function, function notation, linear, polynomial, rational, radical, absolute value, exponential, and logarithmic functions, and transitive property.
ALG2T.21.2: Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations.
ALG2T.21.3: Find solutions to the equations $y = f(x)$ and $y = g(x)$ using the graphing calculator.
ALG2T.21.4: Solve equations for y.
ALG2T.21.5: Apply the properties of multiplicative inverses.
ALG2T.21.6: Apply the properties of exponentials.
ALG2T.21.7: Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations.

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

22. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** [F-IF4]

Objectives:

ALG2T.22.1: Define intercepts, intervals, relative maxima, relative minima, symmetry, end behavior, and periodicity.
ALG2T.22.2: For a function that models a relationship between two quantities, find the periodicity.
ALG2T.22.3: For a function that models a relationship between two quantities, find the end behavior.
ALG2T.22.4: For a function that models a relationship between two quantities, find the symmetry.
ALG2T.22.5: For a function that models a relationship between two quantities, find the intervals where the function is increasing, decreasing, positive, or negative.
ALG2T.22.6: For a function that models a relationship between two quantities, find the relative maxima and minima.
ALG2T.22.7: For a function that models a relationship between two quantities, find the x and y intercepts.

23. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Objectives:

ALG2T.23.1: Define domain, range, relation, function, table of values, and mappings.
ALG2T.23.2: Determine the appropriate domain for a given function.
ALG2T.23.3: Identify functions from information in tables, sets of ordered pairs, and mappings.
ALG2T.23.4: Translate verbal phrases into a function.
ALG2T.23.5: Graph a function on a coordinate plane.
ALG2T.23.6: Arrange data given as ordered pairs into a table and a table of values into ordered pairs.
ALG2T.23.7: Identify the x and y values in an ordered pair.

24. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Objectives:

ALG2T.24.1: Define average rate of change as slope.
ALG2T.24.2: Estimate the rate of change from a graph (rise/run).
ALG2T.24.3: Interpret the average rate of change.
ALG2T.24.4: Calculate the average rate of change.
ALG2T.24.5: Compute the slope of a line given two ordered pairs.
ALG2T.24.6: Identify the slope, given slope-intercept form.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

25. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]

Objectives:
ALG2T.25.1: Define piecewise-defined functions and step functions.
ALG2T.25.2: Graph functions expressed symbolically by hand in simple cases.
ALG2T.25.3: Graph functions expressed symbolically using technology for more complicated cases.

a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]

Objectives:
ALG2T.25a.1: Define square root, cube root, and absolute value function.
ALG2T.25a.2: Graph piecewise-defined functions.
ALG2T.25a.3: Graph step functions.
ALG2T.25a.4: Graph cube root functions.
ALG2T.25a.5: Graph square root functions.
ALG2T.25a.6: Graph absolute value functions.

b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

Objectives:
ALG2T.25bc.1: Define exponential function, logarithmic function, trigonometric function, intercepts, end behavior, period, midline, and amplitude.
ALG2T.25bc.2: Graph logarithmic functions showing intercepts and end behavior.
ALG2T.25bc.3: Graph exponential functions showing intercepts and end behavior.
ALG2T.25bc.4: Graph trigonometric functions showing period, midline, and amplitude.

26. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

Objectives:
ALG2T.26.1: Write a function defined by an expression in different but equivalent forms to reveal different properties of the function.
ALG2T.26.2: Write a function defined by an expression in different but equivalent forms to explain different properties of the function.

27. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Objectives:
ALG2T.27.1: Compare properties of two functions each represented in a different way.
ALG2T.27.2: Identify properties of functions defined algebraically.
ALG2T.27.3: Identify properties of functions defined by verbal description.
ALG2T.27.4: Identify properties of functions defined graphically.
ALG2T.27.5: Identify properties of functions defined numerically in tables.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

28. Write a function that describes a relationship between two quantities.* [F-BF1]

Objectives:
ALG2T.28.1: Write a function that describes a relationship between two quantities. (Linear)
ALG2T.28.2: Write a function that describes a relationship between two quantities. (Exponential)
ALG2T.28.3: Write a function that describes a relationship between two quantities. (Quadratic)

a. Combine standard function types using arithmetic operations. [F-BF1b]
   Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Objectives:
ALG2T.28a.1: Define standard function types as exponential, quadratic, and linear.
ALG2T.28a.2: Combine standard functions by dividing a constant function.
ALG2T.28a.3: Combine standard functions by multiplying a constant function.
ALG2T.28a.4: Combine standard functions by subtracting a constant function.
ALG2T.28a.5: Combine standard functions by adding a constant function.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

29. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

Objectives:
ALG2T.29.1: Recognize even and odd functions from algebraic expressions for them.
ALG2T.29.2: Recognize even and odd functions from their graphs.
ALG2T.29.3: Experiment with various cases of functions and illustrate an explanation of the effects on the graph using technology. (Exponential, Quadratic, Absolute Value).
ALG2T.29.4: Find the value of k given the graphs of $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative);
ALG2T.29.5: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

30. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
    Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Objectives:
ALG2T.30.1: Solve an equation of the form $f(x) = c$ for a simple linear function f that has an inverse.
ALG2T.30.2: Write an expression for the inverse of a simple linear function f of the form $f(x) = c$.
ALG2T.30.3: Apply the substitution principle.
ALG2T.30.4: Solve a multi-step equation.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. ***(Logarithms as solutions for exponentials.)***

31. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

Objectives:
ALG2T.31.1: Define logarithmic and exponential function.
ALG2T.31.2: Recognize the inverse relationship of logarithmic function and exponential functions.
ALG2T.31.3: Calculate the change of base formula for logarithms.
ALG2T.31.4: Recall laws of exponents.
ALG2T.31.5: Apply the properties of logarithms.
ALG2T.31.6: Discuss the appropriateness of the solution.

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

32. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. [F-TF1]

ALG2T.32.1: Define arc length, radian measure, and sector
ALG2T.32.2: Discuss the relationship between arc length and angles.
ALG2T.32.3: Apply the arc length formula.

33. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. [F-TF2]

ALG2T.33.1: Define unit circle, trigonometric functions, periodic functions and radians.
ALG2T.33.2: Apply special right triangles to trigonometric ratios.
ALG2T.33.3: Recall Pythagorean Theorem.
ALG2T.33.4: Demonstrate periodicity of trigonometric functions.

34. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

ALG2T.34.1: Define unit circle, trigonometric functions, periodic functions, trigonometric ratios and radians.
ALG2T.34.2: Apply special right triangles to trigonometric ratios.
ALG2T.34.3: Recall Pythagorean Theorem.
ALG2T.34.4: Demonstrate periodicity of trigonometric functions.
ALG2T.34.5: Discuss reciprocal functions.

Model periodic phenomena with trigonometric functions.

35. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* [F-TF5]

ALG2T.35.1: Define amplitude, frequency, period, vertical and horizontal translation, and midline.
ALG2T.35.2: Calculate amplitude, frequency, period, vertical and horizontal translations, and midline from given data.
ALG2T.35.3: Graph the trigonometric function.
ALG2T.35.4: Demonstrate the standard sine, cosine and tangent function graphs.

Prove and apply trigonometric identities.

36. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. [F-TF8]

Objectives:
ALG2T.36.1: Derive the Pythagorean identity using the equation of the unit circle.
ALG2T.36.2: Recognize the relationship of trigonometric ratios.
ALG2T.36.3: Recall inverse trigonometric functions.

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

37. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. [S-ID4]

ALG2T.37.1: Define mean, standard deviation, normal distribution, normal curve, data sets, and population percentages.
ALG2T.37.2: Calculate the estimated area under the curve.
ALG2T.37.3: Estimate the population percentages.
ALG2T.37.4: Calculate the standard deviation of the data set.
ALG2T.37.5: Draw a normal curve using given data.

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

38. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. [S-IC1]

ALG2T.38.1: Define statistics, random sample, population and parameter.
ALG2T.38.2: Discuss statistics as a process for making inferences about population parameters based on a random sample from that population.
ALG2T.38.3: Apply statistics as a process for making inferences about population parameters based on a random sample from that population.

39. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. [S-IC2]
    Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

ALG2T.39.1: Compare and contrast theoretical models and experimental models.
ALG2T.39.2: Analyze the results.
ALG2T.39.3: Perform the experiment manually, abstractly or using technology.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

40. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. [S-IC3]

Objectives:
ALG2T.40.1: Define sample surveys, experiments, observational studies and randomization.

ALG2T.40.2: Discuss how randomization relates to sample surveys, experiments and observational studies.
ALG2T.40.3: Compare and contrast sample surveys, experiments and observational studies.

41. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. [S-IC4]

Objectives:
ALG2T.41.1: Define population, mean, proportion, margin of error, simulation and random sampling.
ALG2T.41.2: Use data from a sample survey to estimate a population mean or proportion.
ALG2T.41.3: Calculate an estimate of the population mean or proportion.
ALG2T.41.4: Calculate a margin of error through the use of simulation models for random sampling.
ALG2T.41.5: Simulate a model.

42. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. [S-IC5]

Objectives:
ALG2T.42.1: Define randomized experiment, treatments, and parameters.
ALG2T.42.2: Interpret the data.
ALG2T.42.3: Simulate the experiment.
ALG2T.42.4: Compare the treatments of the experiment.

43. Evaluate reports based on data. [S-IC6]

Objectives:
ALG2T.43.1: Analyze the data.
ALG2T.43.2: Determine whether the report accurately portrays the data.

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (*Include more complex situations*.)

44. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

Objectives:
ALG2T.44.1: Describe processes that can be used to make fair decisions.
ALG2T.44.2: Demonstrate a random number generator.
ALG2T.44.3: Generate a table/chart of outcomes.

45. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Objectives:
ALG2T.45.1: Examine the context of given probabilities.
ALG2T.45.2: Interpret decisions and strategies of given probability concepts.
ALG2T.45.3: Interpret expressions.

APPENDIX A
Directions for Interpreting the Minimum Required Content

The illustrations below and on the next page are intended to serve as guides for interpreting the Grades K-12 minimum required content that all students should learn and be able to do in order to be college- and career-ready. Grades K-8 content standards are grouped according to domain and clusters, while Grades 9-12 standards are grouped by conceptual categories, domains, and clusters.

Domains are large groups of related clusters and content standards. Sometimes standards from different domains may be closely related. In the illustration below, the domain is "Number and Operations in Base Ten."

Clusters are groups of related content standards. Due to the fact that mathematics is a connected subject, standards from different clusters may sometimes be closely related. In the example below, the cluster is "Generalize place value understanding for multi-digit whole numbers."

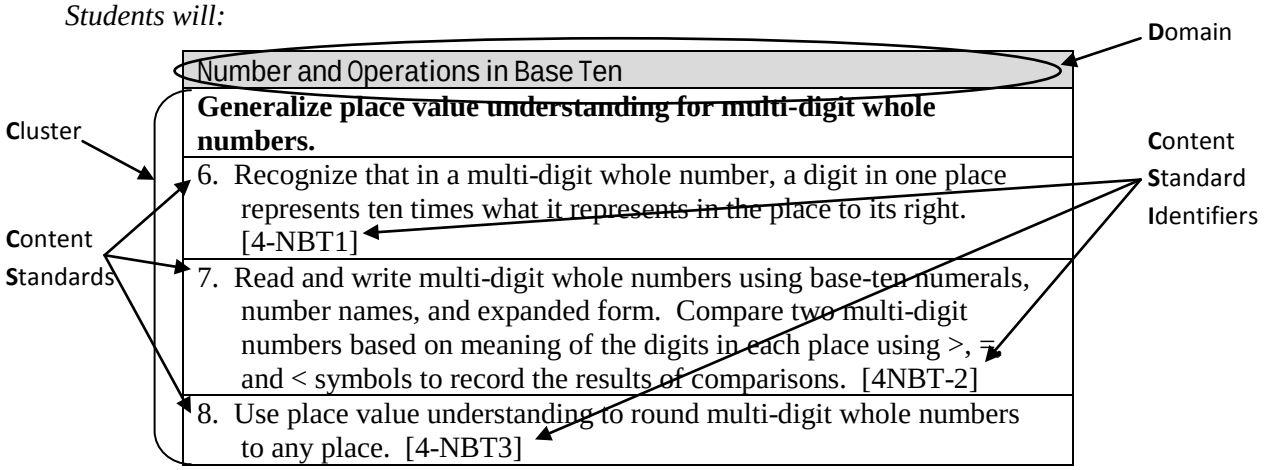
Content Standards are written beneath each cluster as shown in the following illustrations. Standards define what students should understand (know) and be able to do at the conclusion of a course or grade. Content standards in this document contain minimum required content. The order in which standards are listed within a course or grade is not intended to convey a sequence for instruction. Each content standard completes the phrase "*Students will*."

Standards do not dictate curriculum or teaching methods. For example, one topic may appear before a second in the standards for a given grade, but this does not necessarily mean that the first must be taught before the second. A teacher might prefer to teach the second topic before the first topic, or might choose to highlight connections by teaching both topics at the same time. In addition, a teacher might prefer to teach a topic of his or her own choosing that leads, as a by-product, to students reaching the standards for both topics.

Content Standard Identifiers are found in the brackets following each content standard. In the illustration below for Grade 4, this information identifies the student grade level, the national mathematics Common Core State Standard (CCSS) domain, and the CCSS number. For example, the first content standard in the example is followed by content standard identifier [4-NBT1] to indicate the student grade level as fourth (4), the CCSS domain as Number and Operations in Base Ten (NBT), and the CCSS number as one (1).

GRADE 4

Students will:

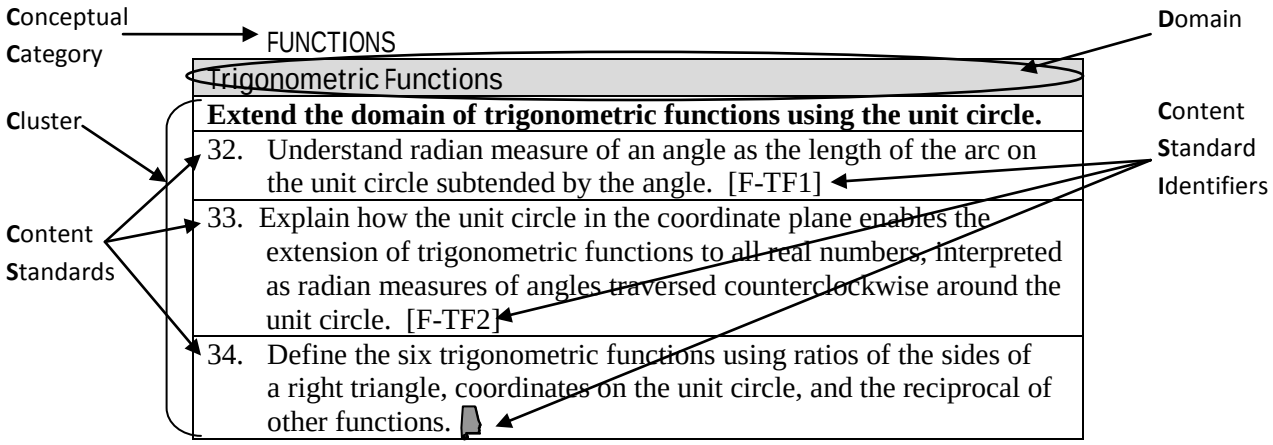


APPENDIX A
Directions for Interpreting the Minimum Required Content

For high school courses, as in the illustration below, the bracketed information identifies the conceptual category by which the standard is grouped, the CCSS domain, and the CCSS number. Conceptual categories are described in the high school mathematics section of this document on pages 67-80. In the illustration below for Algebra II With Trigonometry, the second content standard is followed by content standard identifier [F-TF2] to indicate the CCSS conceptual category as Functions (F), the domain as Trigonometric Functions (TF), and the CCSS number as two (2). Required content added from the 2009 *Alabama Course of Study: Mathematics* is noted at the end of a standard by a state of Alabama symbol (), as shown in the third content standard of the example.

ALGEBRA II WITH TRIGONOMETRY

Students will:



APPENDIX B
Algebraic Concepts (Alabama Occupational Diploma Course)

Algebraic Concepts

Students will:

The Number System

Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers. (7.4)

Objectives:

ALCON.1.1: Define fraction, integer, rational number, absolute value, sum, difference, product and quotient.

ALCON.1.2: Demonstrate proficiency in basic operations on whole numbers and decimals, including basic facts and multi-digit numbers.

ALCON.1.3: Use basic operations to solve problems involving fractions with like and unlike denominators, including mixed numbers.

ALCON.1.4: Use basic operations to solve numerical problems involving integers.

ALCON.1.5: Solve real-world problems involving whole numbers, decimals, fractions, and integers, using the four basic operations.

ALCON.1.6: Determine place values through billions and decimals through ten-thousandths.

ALCON.1.7: Find the absolute value of integer numbers.

ALCON.1.8: Find the square root of perfect squares.

Know that there are numbers that are not rational, and approximate them by rational numbers. (8.1-2)

Objectives:

ALCON.2.1: Define irrational number.

ALCON.2.2: Round whole numbers and decimals to a specified place value.

ALCON.2.3: Know that numbers that are not rational are called irrational.

ALCON.2.4: Convert between fractions and decimals.

ALCON.2.5: Find the square roots of imperfect squares, utilizing the √ key on a four-function calculator.

APPENDIX B
Algebraic Concepts (Alabama Occupational Diploma Course)

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems. (7.1-3)

Objectives:

ALCON.3.1: Define percentage, unit rate, proportion, and ratio.
ALCON.3.2: Convert between decimals and percent.
ALCON.3.3: Solve problems involving percent, including discounts, sales tax, etc.
ALCON.3.4: Identify different forms of notation to symbolize ratios and rates.
ALCON.3.5: Create a ratio or proportion from a given word problem.
ALCON.3.6: Recognize and represent proportional relationships between quantities.
ALCON.3.7: Determine whether or not two ratios are in a proportional relationship through the use of cross products.
ALCON.3.8: Calculate unit rates by using ratios or proportions, including those involving unit pricing and constant speed.
ALCON.3.9: Calculate a proportion for missing information.
ALCON.3.10: Solve a proportion using part over whole equals percent over 100.

Understand the connections among proportional relationships, lines, and linear equations. (8.7-8)

Objectives:

ALCON.4.1: Define slope.
ALCON.4.2: Graph proportional relationships, interpreting the unit rate as the slope of the graph..
ALCON.4.3: Demonstrate how to plot points and graph on the Cartesian plane.
ALCON.4.4: Compute slope of a line using the slope formula and two ordered pairs.
ALCON.4.5: Use rise over run to find the slope between two points plotted on a graph.
ALCON.4.6: Use slope-intercept form to graph the line of an equation on the Cartesian plane.

Expressions and Equations

Use properties of operations to generate equivalent expressions. (7.7-8)

Objectives:

ALCON.5.1: Define linear expression, term, variable, exponent and coefficient..
ALCON.5.2: Simplify an expression by dividing by the greatest common factor.
ALCON.5.3: Simplify expressions with parenthesis by using the distributive property.
ALCON.5.4: Combine like terms in an expression.
ALCON.5.5: Recognize that a variable without a written coefficient is understood to have a coefficient of one.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.(7.9-10)

Objectives:

ALCON.6.1: Define order of operations, inverse operations, and inequality.
ALCON.6.2: Use order of operations to evaluate and simplify numerical and algebraic expressions.
ALCON.6.3: Demonstrate an understanding of the terms *greater than, less than,* and *equal to*.
ALCON.6.4: Identify the symbols $\leq$, $\geq$, $>$, $<$, and $=$.
ALCON.6.5: Solve one- and two-step equations.
ALCON.6.6: Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.
ALCON.6.7: Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables.
ALCON.6.8: Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations.
ALCON.6.9: Determine solutions sets for inequalities and represent them on a number line.

Work with radicals and integer exponents. (8.3-6)

Objectives:

ALCON.7.1: Define exponent, power, and scientific notation.
ALCON.7.2: Know and apply the properties of integer exponents to generate equivalent numerical expressions.
ALCON.7.3: Write a number using scientific notation, and expand a number written in scientific notation.

Functions

Define, evaluate and compare functions. (8.11-13)

Objectives:

ALCON.8.1: Define function, ordered pairs, input and output.
ALCON.8.2: Identify the *x* and *y* values in an ordered pair.
ALCON.8.3: Identify functions from information in tables, sets of ordered pairs, and mappings.
ALCON.8.4: Identify the rule that defines a function.
ALCON.8.5: Find the range of a function when given the domain.
ALCON.8.6: Classify functions as linear or nonlinear by examining their graphs.

Statistics and Probability

Use random sampling to draw inferences about a population. (7.17-18)

Objectives:

ALCON.9.1: Define sample, validity, population, inference, random samplings, statistic, and generalization.
ALCON.9.2: Discuss real-world examples of valid sampling and generalizations.
ALCON.9.3: Predict an outcome of the entire population based on random sampling.
ALCON.9.4: Interpret data from populations, using given and collected data.
ALCON.9.5: Organize collected data, including representing the data graphically.

Investigate chance processes and develop, use, and evaluate probability models. (7.21-24)

Objectives:

ALCON.10.1: Find the probability of a simple event, expressing the probability as ratio, fraction, decimal or percent.
ALCON.10.2: Determine the probability of a compound event.
ALCON.10.3: Determine the theoretical probability of an event.

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.4.2: Choose the scale and the origin in graphs.	**ALG1-B.1.4:** Apply the properties of integer exponents to generate equivalent numerical expressions.	**GEO-A.1.1:** Define angle, circle, perpendicular line, parallel line, line segment, and distance.	**GEO-B.2.1:** Define distance, angle, input, output, plane, translation, and transformations.
ALG1-A.4.3: Interpret units consistently in formulas.	**ALG1-B.1.5:** Know the properties of integer exponents.	**GEO-A.1.3:** Identify angle, circle, perpendicular line, parallel line, line segment, and distance.	**GEO-B.2.3:** Describe transformations as functions that take points in a plane as inputs and give other points as outputs.
ALG1-A.4.4: Choose units consistently in formulas.	**ALG1-B.1.6:** Write numerical expressions involving whole-number exponents.	**GEO-A.3.1:** Define a rectangle, parallelogram, trapezoid, regular polygon, rotation and reflection.	**GEO-B.2.5:** Generate an input output table.
ALG1-A.4.5: Use units as a way to guide the solution of multistep problems.	**ALG1-B.1.7:** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.	**GEO-A.3.2:** Identify which reflections maintain the original properties of the shape.	**GEO-B.2.6:** Measure distance and angle(s) of an image.
ALG1-A.4.6: Use units as a way to understand problems.	**ALG1-B.2.3:** Recognize the properties of exponents.	**GEO-A.3.3:** Demonstrate rotation and reflections on rectangle, parallelogram, trapezoid, regular polygon.	**GEO-B.12.1:** Construct a copy of a segment, copy of an angle, the bisection of a segment , the bisection of an angle, perpendicular line, perpendicular bisector of a line segment, and parallel lines.
ALG1-A.4.7: Convert between units of measurement within the same system.	**ALG1-B.7b.3:** Recognize one or more parts of a linear expression as a single entity.	**GEO-A.3.4:** Distinguish between a rotation and a reflection given an illustration.	**GEO-B.12.3:** Demonstrate the proper use of a geometric construction tools.
ALG1-A.5.1: Define units of measurement.	**ALG1-B.8.3:** Rewrite a linear expression in an alternative form.	**GEO-A.4.1:** Define rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	**GEO-B.17.1:** Define Pythagorean Theorem.

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.5.2: Identify appropriate units of measure to best describe a real-world application.	**ALG1-B.9a.1:** Define factor, quadratic expression and zero product property.	**GEO-A.5.3:** Illustrate figures transformed by a rotation, reflection or translation.	**GEO-B.17.3:** Use Pythagorean Theorem to find the missing side of a right triangle.
ALG1-A.6.2: Determine the level of precision needed for real-world measurements.	**ALG1-B.9a.3:** Use the zero product property to reveal the zeros in the function.	**GEO-A.5.5:** Graph a figure on a coordinate plane.	**GEO-B.17.6:** Identify the parts of a right triangle.
ALG1-A.6.3: Relate how rounding effects the accuracy of the measurement.	**ALG1-B.9b.1:** Define maximum and minimum value.	**GEO-A.6.3:** Illustrate a vertical and horizontal shift on a coordinate plane.	**GEO-B.18.2:** Illustrate congruence and similarity in geometric figures.
ALG1-A.7.1: Define linear, quadratic and exponential functions.	**ALG1-B.9c.1:** Define roots.	**GEO-A.7.1:** Define congruent, corresponding, triangles, and angles.	**GEO-B.18.4:** Solve proportions.
ALG1-A.7.3: Classify an expression as linear, quadratic or exponential from an equation.	**ALG1-B.9c.2:** Find the equation using the distributive property.	**GEO-A.7.3:** Compare angles and sides of two triangles to determine congruency**.**	**GEO-B.19.1:** Define trigonometric ratios for acute angles**.**
ALG1-A.7.4: Classify an expression as linear, quadratic or exponential from a graph	**ALG1-B.9c.3:** Locate and identify the roots on a graph using the *x*-intercepts.	**GEO-A.7.4:** Identify corresponding parts of triangles.	**GEO-B.19.3:** Identify corresponding parts of similar triangles.
ALG1-A.7a.1: Define terms, factors, and coefficients.	**ALG1-B.9c.4:** Take given roots and convert into a one-step equation set equal to zero.	**GEO-A.7.5:** Measure sides and angles of triangles.	**GEO-B.20.1:** Define sine, cosine, and complementary angles.
ALG1-A.7a.5: Identify parts of an expression using mathematical terms (sum, term, product, factor,	**ALG1-B.9d.1:** Define multiplicative inverse.	**GEO-A.8.2:** Determine corresponding parts of triangles.	**GEO-B.20.3:** Find the complement of a given angle.
ALG1-A.15.2: Solve a multi-step equation using the properties, assuming that the original equation has a solution.	**ALG1-B.10.1:** Define monomial, term, binomial, trinomial and polynomial.	**GEO-A.8.3:** State the angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), Theorems.	**GEO-B.21.1:** Create an equation using the given information**.**
ALG1-A.15.3: Use square root and cube root symbols to represent solutions to equations of the form $x^2=p$ p and $x^3=p$, where p is a positive rational number.	**ALG1-B.10.3:** Multiply polynomial expressions (linear).	**GEO-A.8.4:** Interpret specific symbols (hash marks and arcs) on shapes.	**GEO-B.21.2:** Solve equations involving exponents and radicals.

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.15.4: Evaluate square roots and cube roots.	**ALG1-B.10.4:** Subtract polynomial expressions.	**GEO-A.9.1:** Define vertical angle, transversal, parallel lines, alternate interior angles, corresponding angles, perpendicular bisector, line segment, equidistant, and endpoints.	**GEO-B.21.3:** Solve equations using ratios.
ALG1-A.21.1: Understand that the graph of an equation is the solution of an equation.	**ALG1-B.10.5:** Add polynomial expressions.	**GEO-A.9.5:** Identify measures of vertical angle, alternate interior angles, and corresponding angles.	**GEO-B.21.4:** Label parts of a right triangle.
ALG1-A.21.2: Graph a linear equation and use the graph to determine the solution set.	**ALG1-B.11.1:** Define equation, expression, variable, equality and inequality.	**GEO-A.10.1:** Define interior angles of a triangle ,base angles of isosceles triangles, isosceles triangles, midpoint, median, intersection.	**GEO-B.25.1:** Compare the parts of different circles thereby proving similarity.
ALG1-A.21.3: Use a given graph to determine the solution set.	**ALG1-B.11.6:** Compare and contrast equations and inequalities.	**GEO-A.10.5:** Identify measures of interior angles of a triangle, base angles of isosceles triangles, isosceles triangles, midpoint, median.	**GEO-B.25.2:** Identify parts of a circle.
ALG1-A.22.1: Define function, function notation, linear, polynomial, rational, absolute value, exponential, and logarithmic functions, and transitive property.	**ALG1-B.12.1:** Define ordered pair and coordinate plane.	**GEO-A.10.6:** Find the measure of the third interior angle when given the measure of the other two interior angles.	**GEO-B.26.1:** Define inscribed, central, and circumscribed angles, radius, chord, tangent, and diameter.
ALG1-A.22.4: Solve equations for *y*.	**ALG1-B.13.1:** Define systems of equations, constraints, viable solution, and nonviable solution.	**GEO-A.11.1:** Define opposite sides, opposite angles, diagonals, parallelogram, bisector, and converse.	**GEO-B.26.3:** Illustrate radii, chords, diameters, tangents to curve, central, inscribed, and circumscribed angles.
ALG1-A.24.1: Define domain, range, relation, function, table of values, input, and output.	**ALG1-B.13.4:** Determine if a solution to a system of equations or inequalities is viable or nonviable.	**GEO-A.11.5:** Identify measures of opposite sides, opposite angles, or diagonals of parallelograms.	**GEO-B.29.1:** Define arc length, radian measure, and sector.

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.24.4: Identify the equation of a function, given its graph.	**ALG1-B.13.5:** Determine if there is one solution, infinite solutions, or no solutions to a system of equations or inequalities.	**GEO-A.13.1:** Define equilateral triangle, regular hexagon, inscribed.	**GEO-B.29.4:** Illustrate an arc of a circle by constructing the radii of a circle.
ALG1-A.24.6: Recognize that *f(x)* and *y* are the same.	**ALG1-B.14.2:** Identify and isolate a specific variable.	**GEO-A.13.3:** Recognize given figures as inscribed.	**GEO-B.36.1:** Define circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone, oblique, radius, diameter, and height, base.
ALG1-A.25.1: Define function notation.	**ALG1-B.14.3:** Demonstrate the process of solving multi-step equations.	**GEO-A.14.1:** Define dilation and scale factor.	**GEO-B.36.4:** Calculate the circumference of a circle; area of a circle; and volumes of a cylinder, pyramid, and cone.
ALG1-A.25.3: Evaluate function when given *x*-values.	**ALG1-B.17.1:** Define quadratic equation and zero product property.	**GEO-A.14.3:** Apply a scale factor.	**GEO-B.37.1:** Define cylinder, pyramid, cone, and sphere. Define the volume formulas of cylinders, pyramids, cones, and spheres.
ALG1-A.29.1: Define average rate of change as slope.	**ALG1-B.17a.1:** Define completing the square.	**GEO-A.15.1:** Define similarity and proportions.	**GEO-B.37.4:** Calculate the volume of cylinders, pyramids, cones, and spheres.
ALG1-A.29.4: Calculate the average rate of change.	**ALG1-B.17b.1:** Define quadratic formula, factoring, square root, complex number, and real number.	**GEO-A.15.2:** Compare two figures in terms of similarity.	**GEO-B.37.5:** Calculate the area of the base shape**.**
ALG1-A.29.5: Compute the slope of a line given two ordered pairs.	**ALG1-B.17b.3:** Solve quadratic equations by the quadratic formula.	**GEO-A.16.2:** Determine corresponding angles of triangles.	**GEO-B.37.6:** Identify the base shape.
ALG1-A.29.6: Identify the slope, given slope-intercept form.	**ALG1-B.17b.4:** Solve quadratic equations by factoring.	**GEO-A.16.3:** State the AA postulate.	**GEO-B.39.1:** Define two-dimensional objects and three-dimensional objects**.**
ALG1-A.30a.3: Graph linear functions showing intercepts.	**ALG1-B.17b.5:** Solve quadratic equations by taking square roots (e.g., for x^2=49).	**GEO-A.27.1:** Define inscribed and circumscribed circle of a triangle.	**GEO-B.39.4:** Distinguish between two-dimensional and three dimensional objects.

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.37.1: Define linear function and exponential function.	**ALG1-B.18.1:** Define the elimination process and the substitution process for solving systems of equations.	**GEO-A.27.4:** Illustrate inscribed and circumscribed circles of a triangle and quadrilaterals inscribed in a circle.	**GEO-B.40.3:** Identify the relationship of geometric representations to real-life objects.
ALG1-A.37.2: Distinguish between graphs of a line and an exponential function.	**ALG1-B.18.2:** Use the elimination process to solve systems of equations.	**GEO-A.31.2:** Illustrate polygons created by given coordinates on a coordinate plane.	**GEO-B.41.1:** Define density, area, volume.
ALG1-A.37.3: Identify the graph of an exponential function.	**ALG1-B.18.3:** Use the substitution process to solve systems of equations.	**GEO-A.32.1:** Define slope, point slope formula, slope-intercept formula, standard form of a line, parallel lines, and perpendicular lines.	**GEO-B.41.4:** Illustrate a design conflict (e.g., draw a chair and a desk where the chair will not fit under the desk.)
ALG1-A.37b.1: Define constant rate of change as slope.	**ALG1-B.18.4:** Evaluate an expression in two variables for given values.	**GEO-A.32.5:** Illustrate graphically how parallel lines have no common points.	
ALG1-A.37b.2: Subtract each *y*-value in a table of values by its successive *y*-value to determine if the differences are the same, to prove a linear function.	**ALG1-B.18.5:** Evaluate an expression in one variable for given values.	**GEO-A.32.6:** Illustrate graphically how perpendicular lines have only one common point.	
ALG1-A.37b.3: Recognize the calculated difference is the constant rate of change.	**ALG1-B.19.1:** Define system of linear equations.	**GEO-A.32.7:** Find the slope of a given line.	
ALG1-A.39.1: Define a polynomial function and parabola.	**ALG1-B.19.2:** Verify an equation in two variables for a given ordered pair.	**GEO-A.33.1:** Define line segment, distance formula, and ratio.	
ALG1-A.40.1: Define *b* as growth or decay factor in the context of an exponential problem.	**ALG1-B.19.3:** Identify the point of intersection given graphs of two non-parallel lines in the coordinate plane.	**GEO-A.33.4:** Solve equations using the distance formula.	
ALG1-A.40.2: Define *k* as the initial amount in the context of an exponential problem.	**ALG1-B.19.4:** State the formulas for slope-intercept form, point-slope form, and standard form of a line.	**GEO-A.34.1:** Define perimeter, polygons, areas of triangles, area of rectangles, distance formula.	

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.40.3: Define *m* as the rate of change in the context of a linear problem.	**ALG1-B.19.5:** Construct graphs of common relations, including x = constant, y = constant, $y = x$, and $= mx + b$.	**GEO-A.34.3:** Apply the found lengths to the appropriate formulas for area and perimeter.	
ALG1-A.40.4: Define *b* as the initial amount in the context of a linear problem.	**ALG1-B.20.2:** Solve for the variables in a system of equations. (Algebraically).	**GEO-A.34.4:** Use a given illustration to determine area or perimeter.	
ALG1-A.40.6: Recall the slope-intercept form of a linear function.	**ALG1-B.20.3:** Graph a quadratic equation.	**GEO-A.35.1:** Define area, perimeter, regular polygons, inscribed polygons, circumscribed polygons, and vertices.	
ALG1-A.41.1: Define dot plots, histograms, and box plots.	**ALG1-B.20.4:** Graph a linear equation.	**GEO-A.35.5:** Illustrate graphically an inscribed or circumscribed polygon.	
ALG1-A.41.2: Represent data with plots on the real number line, using box plots.	**ALG1-B.20.5:** Identify the point(s) of intersection when given graphs.	**GEO-A.43.1:** Define conditional probability and independence.	
ALG1-A.41.3: Represent data with plots on the real number line, using histograms.	**ALG1-B.23.1:** Define the half-plane as the shaded region.	**GEO-A.43.5:** Interpret a Venn Diagram.	
ALG1-A.41.4: Represent data with plots on the real number line, using dot plots.	**ALG1-B.23.2:** Determine the intersecting shaded region is the solution to the system.	**GEO-A.43.6:** Draw a Venn Diagram.	
ALG1-A.41.5: Plot points using given data.	**ALG1-B.23.3:** Graph the lines of the systems and shade the appropriate region.	**GEO-A.44.1:** Define two-way frequency table, sample space, independence, and conditional probability.	
ALG1-A.42.1: Define center, mean, median, spread, interquartile range, standard deviation, and data set.	**ALG1-B.23.4:** Determine the shaded region is the solution to the inequality.	**GEO-A.44.6:** Collect data for construction of frequency tables.	
ALG1-A.42.5: Calculate the mean of two or more different data sets.	**ALG1-B.23.5:** Graph an inequality and shade the appropriate region.	**GEO-A.45.1:** Define conditional probability and independence.	

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.42.6: Calculate the median of two or more different data sets.	**ALG1-B.23.6:** Determine whether a line should be solid or dotted, depending on the inequality symbol.	**GEO-A.45.4:** Match dependent and independent events to given everyday situations.	
ALG1-A.42.7: Compare the center.	**ALG1-B.27.7:** For a function that models a relationship between two quantities, find the *x* and *y* intercepts.	**GEO-A.46.1:** Define conditional probability, Venn Diagram, and outcomes.	
ALG1-A.42.8: Organize data sets in either increasing or decreasing values.	**ALG1-B.28.1:** Define domain, range, relation, function, table of values, and mappings.	**GEO-A.47.5:** Determine the probability of independent events.	
ALG1-A.43.1: Define outliers.	**ALG1-B.28.2:** Determine the appropriate domain for a given function.	**GEO-A.50.2:** Demonstrate a random number generator.	
ALG1-A.44.1: Define categorical data, two-way frequency table, relative frequency, joint frequency, marginal frequency, and conditional relative frequency.	**ALG1-B.28.4:** Translate verbal phrases into a function.	**GEO-A.50.3:** Generate a table/chart of outcomes.	
ALG1-A.44.7: Summarize categorical data for two categories in two-way frequency tables.	**ALG1-B.28.5:** Arrange data given as ordered pairs into a table and a table of values into ordered pairs.		
ALG1-A.44.8: Analyze data from tables.	**ALG1-B.30b.1:** Define square root, cube root, and absolute value function.		
ALG1-A.45.1: Define scatter plot.	**ALG1-B.30b.6:** Graph absolute value functions.		
ALG1-A.45.4: Construct a scatter plot using given data.	**ALG1-B.30c.1:** Define exponential function, logarithmic function, trigonometric function**,** intercepts, end behavior, period, midline and amplitude.		
ALG1-A.45a.5: Find the equation of a line given data points.	**ALG1-B.31a.5:** Use factoring to solve a quadratic equation.		

Appendix C
Alabama Occupational Diploma Objectives

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.45a.6: Develop a table of values from data on a graph.	**ALG1-B.31b.1:** Define exponential growth and decay.		
ALG1-A.45b.1: Define fit of a function and residuals.	**ALG1-B.31b.4:** Classify an exponential function as growth or decay.		
ALG1-A.45c.1: Define linear function and scatter plot.	**ALG1-B.31b.6:** Apply properties of operations to calculate numbers in any form converting between forms as appropriate.		
ALG1-A.45c.2: Write the equation of a line given two points.	**ALG1-B.32.5:** Identify properties of functions defined numerically in tables.		
ALG1-A.45c.3: Find the slope of a line given two points.	**ALG1-B.33.3:** Write a function that describes a relationship between two quantities. (Linear)		
ALG1-A.46.2: Understand that the *y*-intercept is the initial amount in the context of the data.	**ALG1-B.33a.1:** Define explicit expressions and recursive process.		
ALG1-A.46.3: Understand that rate of change in the context of the data is the label of the *y-axis* divided by the label of the *x-axis*.	**ALG1-B.33b.1:** Define standard function types as exponential, quadratic, and linear.		
ALG1-A.47.1: Define correlation coefficient.	**ALG1-B.33b.4:** Combine standard functions by subtracting a constant function.		
ALG1-A.47.3: Compute (using technology) the correlation coefficient of a linear fit.	**ALG1-B.33b.5:** Combine standard functions by adding a constant function.		
ALG1-A.49.1: Define subsets, sample space, outcomes, union, intersection, and complement.	**ALG1-B.36.1:** Define inverse function.		
ALG1-A.49.4: Describe events as unions of other events (“or”).	**ALG1-B.36.5:** Find inverse functions for a function. (Linear)		

Algebraic Essentials A (♦)	Algebraic Essentials B (◊)	Geometry Essentials A (♦)	Geometry Essentials B (◊)
ALG1-A.49.5: Describe events as intersections of other events ("and").	**ALG1-B.36.6:** Apply the substitution principle.		
ALG1-A.49.6: Describe events as complements of other events ("not").	**ALG1-B.36a.2:** Write an expression for the inverse of a simple linear function *f* of the form *f(x) = c*.		
ALG1-A.50.1: Define probability, ratio, simple event, compound event, and independent event.	**ALG1-B.36a.3:** Apply the substitution principle.		
ALG1-A.50.3: Determine the probability of an independent event.	**ALG1-B.37a.2:** Subtract each *y*-value in a table of values by its successive *y*-value to determine if the differences are the same, to prove a linear function.		
ALG1-A.50.5: Identify the probability of an event that is certain as 1 or impossible as 0.	**ALG1-B.37c.1**: Define exponential growth and decay.		
ALG1-A.50.6: Solve word problems involving probability.	**ALG1-B.38.1:** Define arithmetic sequence, geometric sequence, and input-output pairs.		
ALG1-A.50.7: Use proportional relationships to solve multi-step ratio and percent problems.	**ALG1-B.38.5:** Write an equation of a line, given a chart.		
	ALG1-B.38.6: Write an equation of a line, given a graph.		
	ALG1-B.38.7: Write an equation of a line given two ordered pairs.		

GLOSSARY

Absolute value. A number's distance from zero on the number line. The absolute value of a number *n* is denoted by $|n|$.

Algebraic model. An expression that contains one or more numbers, one or more variables, and one or more arithmetic operations.

Algorithm. A set of steps used to carry out a computation.

Alternate interior angle. Two congruent interior angles that lie on different parallel lines and on opposite sides of a transversal.

Amortization table. A time table or schedule that gives a breakdown of monthly payments into principal and interest.

Amplitude. Half the distance between the minimum and maximum values of the range of a periodic function with a bounded range.

Analytical methods. A mathematical way to solve problems without repetitive attempts to approximate an answer.

Angle. Formed by two rays with a common endpoint called the vertex.

Angle bisector. A ray that divides an angle into two congruent angles.

Application-based problem. A problem that applies mathematical terminology and formulas to real-life situations.

Approximation. An estimation of a number or rounding a number to its nearest place value

Area. The number of square units that covers a closed figure.

Arithmetic operations. Addition, subtraction, multiplication, and division.

Arithmetic sequence. A sequence of numbers that has a constant difference between every two consecutive units.

Associative Property of Addition. If *a*, *b*, and *c* are real numbers than $a + (b + c) = (a + b) + c$. The same terms in the same order added in different groupings yield the same results.

Associative Property of Multiplication. If *a*, *b*, and *c* are real numbers then $(ab)c = a(bc)$. . The same factors in the same order multiplied in different groupings yield the same results.

Base angles. Angles formed by the base and one leg in isosceles triangles.

Bisect. To divide into two equal sections or two equal halves.

Boundary. A border that encloses a space or an area.

Box-and-whisker plot. A graph that shows the distribution of a set of data along a number line, dividing the data into four parts using the median and quartiles.

Cartesian plane. The coordinate plane formed by two perpendicular number lines intersecting at the origin (0,0).

Center. The point inside the circle, from which all points on the circle are the same distance.

Chords. A line segment on the interior of a circle with both its endpoints lying on the circle.

Circle. The locus of all points that are at an equal distance from a given point (on the plane) called the center.

Circle graph. A graph in the form of a circle that is divided into sectors, with each sector representing a part of a set of data.

Coefficient. The numerical part of an algebraic term. For example, in 6xy, the 6 is the coefficient.

Combinations. An arrangement of objects in which order does not matter.

Common factor. A factor shared by all terms in an algebraic expression.

Common ratio. For a geometric sequence or geometric series, the common ratio is the ratio of a term to the previous term. This ratio is usually indicated by the variable *r*.

Commutative Property of Addition. If a and b are real numbers, then $a + b = b + a$.

Commutative Property of Multiplication. If a and b are real numbers, then $ab = ba$.

Completing the square. The process of converting a quadratic equation into a perfect square trinomial by adding or subtracting terms on both sides.

Complex number. A number that can be expressed in the form $x + yi$, where x and y are real numbers

Complimentary angles. Two acute angles whose sum is a right angle (90°).

Composite number. A positive integer greater than 1 that is not a prime number.

Compound events. An event consisting of two or more simple events.

Compound interest. The interest paid on both the principal and the interest earned so far on that principal.

Conditional probability. The probability of an event (A), given that another (B) has already occurred.

Cone. A three-dimensional figure that has one circular base and one vertex.

Congruent. Having the same size and shape.

Constant term. An expression or equation has a fixed value and does not contain variables.

Constraints. Conditions given to variables often expressed as linear inequalities.

Converse. Switching the hypothesis and conclusion of a conditional statement.

Conversion scale. A scale indicating the relationship between two different units of measurement.

Coordinate plane. A plane formed by the intersection of a horizontal number line with a vertical number line.

Coordinates. In a two-dimensional coordinate plane, the pairs of numbers which specify the position or location of a point or of an object.

Correlation coefficient. A measure of the interdependence of two random variables that ranges in value from -1 to +1, indicating perfect fit.

Corresponding angle. Two congruent angles, both lying on the same side of the transversal and situated the same way on two different parallel lines.

Cosine. The ratio of length of the adjacent side to the length of the hypotenuse in a right triangle.

Cross product. The method for solving a proportion.

Cube root. A number or quantity that, when multiplied by itself twice, equals a given number or quantity

Cylinder. A three-dimensional geometric figure that has two congruent and parallel bases.

Decay factor. The base b in the exponential function $y = ab^x$, where $0 < b < 1$.

Depreciation. A decrease or loss in value, as because of age, wear, or market conditions.

Derive. To arrive at by reasoning; deduce or infer.

Diagonal. A line segment connecting two non-adjacent vertices of a polygon.

Diameter. A line segment that passes through the center of a circle with both its endpoints lying on the circle.

Dilation. A transformation in which a geometric figure is enlarged or reduced with respect to a given point (center) and a given scale factor with corresponding lines in the original figure and the dilation being parallel.

Direct variation. A linear function that can be expressed in the form $y = kx$, where $k \neq 0$.

Distance. The length between two points.

Distributive Property of Multiplication over Addition. If a and b are real numbers, then, $a(b+c) = ab + ac$, or $(a+b)c = ac + bc$.

Domain. The set of all first coordinates (x) in the ordered pairs of a relation.

Dot plots. A set of data is represented by using dots over a number line.

Elimination method. The process of eliminating one of the variables in a system of equations using addition or subtraction in conjunction with multiplication or division and solving the system of equations.

End behavior. The appearance of a graph as it is followed farther and farther in either direction.

Equality. A state in which two things or values are always equal.

Equations. A mathematical sentence that uses the equal sign (=) to show that two expressions are equal.

Equidistant. Equal distance between two points.

Equilateral triangle. A triangle with all sides congruent.

Explicit expression. A mathematical phrase that combines numbers and/or variables using mathematical operations.

Explicitly-defined sequence. A formula that allows direct computation of any term for a sequence a_1, a_2, a_3, . . . , a_n, ...

Exponential decay. Occurs when a quantity decreases by the same proportion r in each time period t.
Exponential function. A function of the form $y = ab^x$, where both a and b are greater than 0 and b is not equal to 1.

Exponential growth. Growth that occurs when a quantity increases by the same proportion r in each time period t.

Exponential models. Situations that can be modeled well by the rules of the form $y = a \cdot b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$. They construct and use data tables, graphs and equations to describe and solve exponential problems.

Factor theorem. The theorem that establishes the connection between the zeros and factors of a polynomial.

Factor tree. A hierarchical structure used to represent the prime factors of a number.

Factorials. The produce of a given integer and all smaller positive integers. The factorial of n is written $n!$

Factoring. A method of writing numbers as the product of their factors or divisors.

Frequency distribution. A tabulation of the values that one or more variables take in a sample.

Frequency table. A table that lists items and uses tally marks to record and show the number of times they occur.

Function. A relation in which each element of the domain is paired with exactly one element of the range; a relationship between two quantities in which one quantity depends on the other.

Function notation. A rule written in function notation uses the symbol $f(x)$ in place of y.

Geometric sequence. A sequence in which each term after the first term *a* is obtained by multiplying the previous term by a constant *r*, called the common ratio.

Golden ratio. Sometimes called the golden mean; the number $\frac{1+\sqrt{5}}{2}$, or about 1.61803. The golden ratio arises in many settings, particularly in connection with the Fibonacci sequence.

Graphical representations. Models represented by a graph.

Graphs. A picture that represents data in an organized manner; a line or curve drawn on a number line or coordinate plane by joining the points that represent certain ordered pairs.

Greatest common factor (GCF). The largest factor common to two or more numbers or terms.

Histograms. A bar graph that shows how frequently data occur within certain ranges or intervals. The height of each bar gives the frequency in the respective interval.

Identity property of addition. For all real numbers *a*, there exists a real number 0, such that $a+0=a$ and $0+a=a$. The identity element for addition is 0.

Identity Property of Multiplication. For all real numbers *a*, there exists a real number 1, such that $a \cdot 1 = 1 \cdot a = a$. The identity element for multiplication is 1.

Independent event. Events in which the outcome of one event does not affect the outcome of the other events.

Independent variable. A variable in an equation that may have its value freely chosen without considering values of any other variable.

Inequality. A mathematical expression that uses symbols such as $\geq$, $\leq$, $<$, or $>$ to compare two quantities.

Infinity. A "number" which indicates a quantity, size, or magnitude that is larger than any real number. The number infinity is written ∞. Negative infinity is written $-\infty$.

Input. In a function, the *x* value.

Input-output pairs. Ordered pairs

Inscribed angle. The angle formed by two chords that meet at the same point on a circle.

Inscribed polygon. A polygon placed inside a circle so that all the vertices of the polygon lie on the circumference of the circle.

Integers. All whole numbers and their opposites.

Intercepts. The point at which a curve intersects an axis.

Intersect. Where two lines cross each other at a single point.

Intervals. The set of all real numbers between two given numbers. The two numbers on the ends are the endpoints. The endpoints might or might not be included in the interval depending whether the interval is open, closed, or half-open (same as half-closed).

Intraquartile range. The distance between numeric values representing the 25^{th} and the 75^{th} percentiles.

Inverse function. The function obtained by switching the *x*- and *y*-variables in a function. The inverse of function *f* is written f^{-1}.

Inverse variation. A variation in which the variable *y* varies inversely as *x*.

Isosceles triangle. A triangle with two equal sides.

Joint variation. The same as direct variation, with two or more quantities.

Key features. Sometimes called a legend. Used to identify the number of categories present in a graph. On a pictograph a key tells how many each picture stands for.

Law of cosines. An equation relating the cosine of an interior angle and the lengths of the sides of a triangle. Example $c^2=a^2+b^2 - 2ab\ cosC$.

Law of sines. Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides. $\text{Law of Sines}\quad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ 

Least common multiple (LCM). The smallest integer that is divisible by each of two or more integers.

Line of best fit. A line that closely approximates a set of data.

Line symmetry. The property of a figure that identifies half of the figure as a reflection across a line of the other half of the figure.

Linear equation. An equation in the form of $Ax+By=C$, where $A\neq0$ and $B\neq0$. The graph of a linear equation is a straight line.

Linear function. A function whose linear pairs satisfy a linear equation.

Logarithmic function. The inverse of an exponential function**.**

Maximum value. The *y*-coordinate of the vertex of the quadratic function $f(x)= ax^2 + bx + c = 0$, where $a<0$.

Measure of variation. A measure that describes how spread out or scattered a set of data is; also known as measures of dispersion or measures of spread.

Median. The middle number in a sorted list of numbers.

Midpoint. The point halfway between the endpoints of a line segment.

Minimum value. The *y*-coordinate of the vertex of the quadratic function $f(x)= ax^2 + bx + c = 0$, where $a>0$.

Multiplicative inverse. One of a pair of numbers whose product is 1.

Multiplicative Inverse Property. For every nonzero real number *a*, there is exactly one number $\frac{1}{a}$ such that $a(\frac{1}{a}) = 1$ and $\frac{1}{a}(a) = 1$.

Non-linear relationship. When the dependent variable is not proportional to the independent variable.

Numerical methods. In mathematics, methods of approximate solution of mathematical problems through the performance of a finite number of elementary operations on numbers.

Oblique. Slant.

Ordered pairs. A pair of numbers used to locate a point on a coordinate plane.

Output. The results of a probability experiment or event.

Parallel lines. Lines lying in the same plane. Parallel lines have the same slope.

Parallelogram. A quadrilateral with parallel opposite sides.

Parameter. A value that is already built into a function.

Percent. The number of parts out of 100.

Perimeter. The sum of the lengths of the sides of a polygon

Period. The horizontal distance required for the graph of a periodic function to complete one cycle.

Permutations. An arrangement of objects in which order is important.

Perpendicular bisector. In a triangle, a line, segment or ray that passes through the midpoint of a side and is perpendicular to that side.

Perpendicular lines. Lines that intersect and form right angles.

Piece-wise function. A function *f(x)* that is given by different expressions on various intervals.

Polygon. A simple, closed, many-sided figure.

Polynomial function. Polynomial functions are functions that have this form:
$f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$. The value of n must be an non-negative integer.

Polynomials. The sum or difference of terms which have variables raised to positive integer powers and which have coefficients that may be real or complex.

Precision. The precision of any measurement depends on the smallest unit available on the measuring tool.

Prime number. A positive integer which has only 1 and the number itself as factors.

Probability. A ratio that measures the chances of an event occurring.

Proportion. An equation in the form $\frac{a}{b} = \frac{c}{d}$ that states that two ratios are equal.

Pyramid. A polyhedron with a polygonal base and lateral faces that taper to an apex. A pyramid with a triangular base is called a tetrahedron.

Pythagorean Theorem. A relationship in a right triangle with legs *a* and *b* and hypotenuse *c* for which $a^2+b^2=c^2$.

Quadratic functions. A function in the form $f(x) = ax^2+bx+c$, where *a*, *b*, and *c* are real numbers and $a \neq 0$.

Quadratic formula. A formula for the roots of a quadratic equation; $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Quadratic term. In the equation $f(x) = ax^2+bx+c$, ax^2 is the quadratic term.

Quadrilateral. A polygon with four sides.

Radius. In a circle, any segment with endpoints that are the center of the circle and a point on the circle.

Range of data. The difference between the greatest and least values in a set of data.

Range of function. The set of all output values or the *y*-values of a function or a relation.

Ratio. A comparison of two quantities using division.

Rational functions. A function that can be written as a polynomial divided by a polynomial; an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

Rational numbers. Any number that can be expressed as an integer, a ratio between two integers where zero does not serve as the denominator, or a repeating or terminating decimal.

Real number. Consists of the natural numbers, whole numbers, integers, rational, and irrational numbers.

Recursive formula. Used to determine the next term of a sequence using one or more of the preceding terms.

Reflection. A transformation representing a flip of a figure over a point, line or plane; a transformation that creates a mirror image of a given function.

Regular hexagon. A polygon with six equal sides and six equal angles.

Regular polygon. A polygon with congruent sides and congruent angles.

Relation. A set of input and output values, usually represented in ordered pairs.
Relative frequency. The frequency of the event divided by the total number of data points.

Relative maxima. The largest or the greatest value in a data set.

Relative minima. The smallest or the least value in a data set.

Residuals. The difference of the sample and the estimated function value.

Root of an equation. The value(s) of a variable that make the equation true.

Rotation. A transformation in which a geometric figure rotates around a point without affecting its size or shape.

Rotational symmetry. The property of a figure that identifies a rotation of the figure about a point of the same figure.

Sample space. The set of all possible outcomes of an event.

Scalene triangle. A triangle with no congruent sides.

Scatter plot. A set of data graphed as ordered pairs in a coordinate plane.

Sequence. A list of numbers in a particular order.

Set notation. A shorthand used to write sets, often sets with an infinite number of elements; for example $\{x : x > 0\}$.

Set of outcomes. The set of the results of probability experiments or events.

Shape. The form or outline of a two-dimensional figure or a three-dimensional object.

Simple event. One event.

Sine. In a right triangle, the ratio of the length of the leg opposite an acute angle of the triangle to the length of the hypotenuse. Sin = $\frac{opposite}{hypotenuse}$

Slope. The measure of steepness of a line.

Slope-intercept form of a line. $y = mx + b$, where m is the slope and b is the y-intercept**.**

Sphere. A three-dimensional solid consisting of all points equidistant from a given point.

Square root. A nonnegative number that must be multiplied times itself to equal a given number. The square root of x is written $\sqrt{x}$ or $x^{½}$.

Standard form of a line. $Ax + By = C$, where $A > 0$ and, if possible, A, B, and C are real numbers and A and B are not both zero.

Step function. A function that has a graph resembling a staircase.

Substitution method. A method of solving a system of equations in which variables are replaced with known values or algebraic expressions.

Symmetry. A balanced arrangement of all parts of a figure on opposite sides of a point, line or plane.

System of equations. Two or more equations containing common variable(s).

Table. A chart that organizes data in rows and columns to show facts and figures.

Tangent. A line in the plane of a circle that intersects the circle in exactly one point.

Term. The monomials that make up a polynomial; each number in a sequence or series.

Terminating decimal. A decimal number that ends in all zeros.

Three-dimensional. Having length, width and depth.

Transformation. A change in the position, size, or shape of a figure.

Transitive property. If $a=b$ and $b=c$, then $a=c$.

Translation. A transformation in which a geometric figure slides from one location to another without affecting its size and shape.

Transversal. A line that cuts two or more other lines in the same plane.

Triangle. A closed plane figure with three straight sides.

Triangle Inequality Theorem. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Trigonometric function. The six functions: sine, cosine, tangent, cosecant, secant, and cotangent. These functions can be defined several different ways.

Trigonometric ratios. The types of ratios that compare the length of the sides of a right triangle.
$\text{Sin } A = \frac{opposite}{hypotenuse}$ $\text{Cos } A = \frac{adjacent}{hypotenuse}$ $\text{Tan } A = \frac{opposite}{adjacent}$

Two-dimensional. A term used to describe plane figures in which only the length and width can be measured.

Uniform probability. A type of probability distribution in which all outcomes are equally likely.

Union. The set formed by combining all the elements of two or more sets; shown by the symbol.

Vertex. The point common to both sides of an angle.

Volume. The space occupied by an object; formula is $v=lwh$.

X-axis. The horizontal number line in a coordinate plane.

Y-axis. The vertical number line in a coordinate plane.

Zero Product Property. A property that states that if the product of two or more factors is zero, then at least one of the factors must be zero.

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