LESSON 12

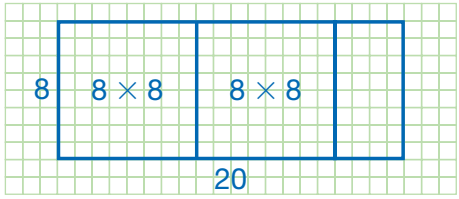
Extending Factors and Multiples to GCF and LCM

UNDERSTAND You can use area models or grids to find the **greatest common factor (GCF)** of two numbers.

Find the GCF of 8 and 20.

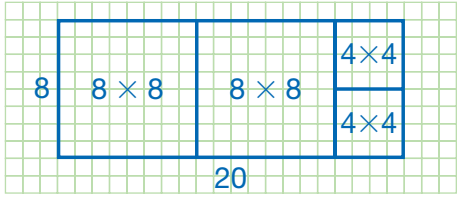
1

Think of 8 and 20 as the dimensions of a rectangle.
To find the GCF using models, find the dimensions of the largest square that can tile the entire rectangle with no gaps or overlaps.
The largest square that fits inside the rectangle is a square against one side with length 8 units long. Two 8 by 8 squares fit inside the rectangle. However, they do not fill the rectangle.



2

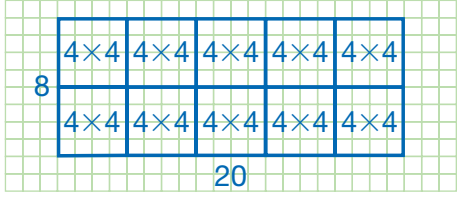
The largest square that fits inside the remaining rectangle is a 4 by 4 square. Two 4 by 4 squares fit and completely fill the original rectangle.



Since the original rectangle is now filled, 4 is the GCF.

3

Notice that the entire 8 by 20 rectangle can be filled with the 4 by 4 squares.



▶ The GCF of 8 and 20 is 4.

Connect

Find the GCF of 8 and 20.

1 List all the **factors** of each number.

Factors of 8: 1, 2, 4, 8

Factors of 20: 1, 2, 4, 5, 10, 20

2 Find the common factors of 8 and 20.

Underline the common factors.
These are the factors in both lists.

Factors of 8: 1, 2, 4, 8

Factors of 20: 1, 2, 4, 5, 10, 20

3 Find the GCF.

Find the greatest factor that appears in both lists.

Factors of 8: 1, 2, 4, 8

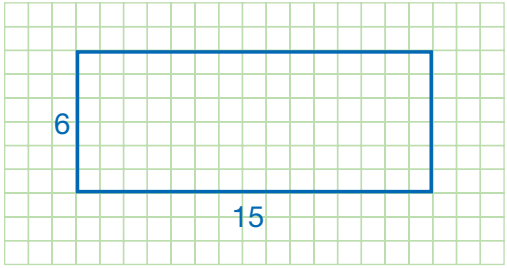
Factors of 20: 1, 2, 4, 5, 10, 20

4 is the greatest factor that appears in both lists.

▶ The GCF of 8 and 20 is 4.

MODEL

Use an area model to find the GCF of 6 and 15.



EXAMPLE A The GCF of two numbers is 3. The two numbers are between 35 and 46. The greater number has two more factors than the lesser number. What are the numbers?

1 Decide which of the numbers between 35 and 46 have 3 as a factor.

Is 3 a factor of:

36? Yes	41? No
37? No	42? Yes
38? No	43? No
39? Yes	44? No
40? No	45? Yes

2 List the factors of 36, 39, 42, and 45.

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

39: 1, 3, 13, 39

42: 1, 2, 3, 6, 7, 14, 21, 42

45: 1, 3, 5, 9, 15, 45

3 Which pairs of numbers have 3 as the GCF?

36 and 39? Yes

36 and 42? No

36 and 45? No

39 and 42? Yes

39 and 45? Yes

42 and 45? Yes

4 The greater number has two more factors than the lesser number.

Does 39 have 2 more factors than 36? No

Does 42 have 2 more factors than 36? No

Does 45 have 2 more factors than 36? No

Does 42 have 2 more factors than 39? No

Does 45 have 2 more factors than 39? Yes

Does 45 have 2 more factors than 42? No

▶ The numbers are 39 and 45.

TRY

Find the GCF of 12 and 18.

EXAMPLE B Use the **distributive property** to express the sum below.

$54 + 12$

1 Find the GCF of 54 and 12.

List the factors of 54 and 12.

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Factors of 12: 1, 2, 3, 4, 6, 12

The GCF of 54 and 12 is 6.

2 Write each addend with 6 as a factor.

$54 = 6 \times 9$

$12 = 6 \times 2$

3 Rewrite the expression as a sum with the factors.

$54 + 12 = (6 \times 9) + (6 \times 2)$

4 Use the distributive property.

Use the GCF as the factor that is distributed to each term in the sum.

$(6 \times 9) + (6 \times 2) = 6(9 + 2)$

▶ $54 + 12 = 6(9 + 2)$

CHECK

How do you know that the equation $54 + 12 = 6(9 + 2)$ is true?

EXAMPLE C Find the **least common multiple (LCM)** of 6 and 9.

1 List some **multiples** of each number.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54

Multiples of 9: 9, 18, 27, 36, 45, 54

2 Find the common multiples.

Underline the common multiples. These are the multiples found in both lists.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54

Multiples of 9: 9, 18, 27, 36, 45, 54

3 Find the LCM.

Find the least multiple that appears in both lists.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54

Multiples of 9: 9, 18, 27, 36, 45, 54

18 is the least multiple that appears in both lists.

▶ The LCM of 6 and 9 is 18.

DISCUSS

How can you choose when to stop while making a list of multiples to find the LCM?

Riddle It!

Solve the Riddles

1. The GCF of two numbers less than or equal to 12 is 2. Their LCM is 20. What are the numbers?

2. The LCM of two numbers less than or equal to 12 is 36. The GCF of the numbers is 3. What are the numbers?

3. The GCF of two numbers is 4. The numbers are between 10 and 20. What are the numbers?

4. The LCM of two numbers less than or equal to 12 is 30 more than 12. What are the numbers?

5. The GCF of two numbers less than 100 is 12. The difference between the numbers is 36. The greater number is a multiple of 10. What are the numbers?

6. Write your own GCF riddle. Exchange your riddle with classmates and solve.

7. Write your own LCM riddle. Exchange your riddle with classmates and solve.

Practice

Use area models or grids to find the GCF of each pair of numbers.

1. 4, 10

2. 6, 12

3. 9, 15

HINT The lesser number can always form at least one square.

Find the GCF of each pair of numbers.

4. 5, 20

5. 9, 24

6. 7, 10

7. 18, 42

8. 14, 35

9. 15, 60

10. 24, 40

11. 25, 45

12. 16, 18

Find the LCM of each pair of numbers.

13. 4, 12

14. 5, 7

15. 2, 10

16. 6, 8

REMEMBER The first multiple of a number is the number itself.

17. 3, 5

18. 4, 7

19. 5, 6

20. 4, 9

21. 8, 12

22. 3, 10

23. 9, 12

24. 2, 11

Use the distributive property to express each sum with the GCF factored out.

25. 45 + 30

26. 16 + 28

27. 32 + 56

28. 24 + 39

29. 50 + 75

30. 35 + 20

Solve.

31. Consider the numbers between 20 and 30. Which number has the greatest number of factors?

32. What is the GCF of 12, 20, and 36?

33. What is the LCM of 3, 4, and 8?

34. Find the GCF and the LCM of 9 and 12.

35. Tristan has 45 apples and 20 pears that he is putting into gift baskets. Each basket will have the same number of apples and pears. What is the greatest number of baskets Tristan can make with no fruit left over? Explain.

36. At Poultry Paradise, turkey burgers are sold in packages of 8. Whole-grain buns are sold in packages of 6. What is the least number of turkey burgers and buns Carly can buy to have an equal number of each? Explain.

37. DEMONSTRATE What is the least common factor of any pair of numbers? Explain.

38. ANALYZE Why are you able to find a greatest common factor but not a greatest common multiple?